AN ECONOMETRIC MODEL OF INDUSTRY, PROFITS, AND TATONNEMENT ADJUSTMENT

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ABSTRACT

This study presents a quantitative analysis of one of the main forces in an economy, disaggregated short term profits, and of the process whereby the system adjusts itself to the temporary equilibrium indicated by such forces, a generalised tatonnement.

Quarterly ten-equation econometric models explaining industry behaviour and profits are developed from a basic industry model for ten mutually exclusive and exhaustive industries. These models are connected with each other and with the whole by a number of linkages and by being embedded in a skeletal economy model.

The system is solved at two levels. Firstly the industry models are solved individually for given values of the linking variables; the results are used to choose between alternative specifications of the models and to assess the adequacy of the formulation adopted. Secondly the whole system is solved iteratively by solving the industry models for some given trial values of the linking variables, using these solutions to derive new trial values, and repeating the process until these values converge; the results are used to assess the efficacy of the tatonnement process.

The results indicate that the models proposed are good predictors of disaggregated short term profits and that the tatonnement process used produces rapid convergence to a consistent equilibrium. It is also suggested from the discrepancy between the tatonnement (quasi-competitive) and actual (imperfectly competitive) solutions that the capitalist system is inefficient in that it produces too much.

CHAPTER 1 INTRODUCTION

This chapter presents the aims, background, and nature of the study: an econometric model of industry, profits, and tatonnement adjustment.

1-1 Aims

Economics is concerned with scarcity: the allocation of scarce resources amongst uses. Its aim is the negation of scarcity, or the achievement of surplus, of profit. Profit in its widest sense is thus a measure of the degree of negation of scarcity; it is existential to economics. More specifically, economics seeks to negate scarcity through the allocation of productive resources amongst productive sectors; sectoral profits thus provide a fundamental force in economics. This applies equally whoever receives these profits, be it society or the capitalist; as Lange [1936] has shown, the realistic accounting for profit is not only possible but essential to a rational socialist economy. Keynes [1930, p. 140] was not concerned with particular systems when he wrote that 'profits (or losses) having once come into existence become . . . a cause of what subsequently ensues; indeed, the mainspring of change in the existing economic system'.

Economic theory is the synthesis of economic forces;

it is representation of these forces by a system of synthetic relations (equations or inequalities) and the determination of the resultant of these forces by the solution of the relations. Such a model, and thus the basis of economic theory, may be considered inadequate unless it also shows by what means the economic forces arrive at the solution, or in effect solve the equilibrium relations. This vital link between the real and the synthetic is provided by the dialectic of Walras' [1954, p. 520] concept of tatonnement[1]: 'a theory of the process by which the market mechanism solves the equilibrium equations . . . , not as a rational, sentient, entity but rather as a blind mechanism so constituted that it automatically makes trial and error adjustments towards equilibrium'.

These twin ideas form the basis of this study: a quantitative analysis of the fundamental motivating forces in an economy, sectoral profits, and the process whereby the economy adjusts itself to the momentary equilibrium indicated by these forces, a generalised tatonnement; it takes the form of a short term sectoral econometric model.

1-2 Background

The general subject matter on which this study is based may be divided into three (rather vague) areas: econometric models (methodology and relevant actual models), profits, and tatonnement. This section presents a brief survey of

1. The term 'tatonnement' does not translate directly, and is retained (with the accent omitted for typographical reasons); the quotation is from the translator's note on the term.

some of the more relevant work in these areas, together with a discussion of extensions and implications where applicable. Work on more specific matters is treated in context.

Models The methodology of econometric models is of basic importance to this study, but as it is on the whole a widely accepted and documented field, the treatment here is limited to formal definition; the general interpretation follows that of Christ [1966]. Econometrics, a term coined by Frisch in 1926 to cover virtually all quantitative economics, both inductive and deductive, is used in its more modern (inductive) sense: defined by Samuelson et al [1954, p. 142] as 'the quantitative analysis of actual economic phenomena based on the concurrent development of theory and observation, related by appropriate methods of inference'. Koopmans [1953, p. 29] defines a model as 'a set of structures', where 'structures' comprise structural equations (those explaining economic, institutional, or technical phenomena) and statements about the distribution functions of latent variables (shift variables that are not directly observable - 'error terms'). An equivalent definition of a structure given by Christ [1966, p. 153] is 'a set of autonomous relationships sufficient to determine uniquely the conditional probablity distributions of the endogenous variables, given the values of the exogenous variables'. Thus estimation converts a model into a structure; frequently however the general term (model) is used for the particular (structure).

There are a number of macroeconometric models, originating from Tinbergen's [1939] pioneer work. This makes explicit the fundamental method followed by all subsequent work: an overall structure is postulated, and split into a number of directly causal economic relations expressing a priori ideas, which are estimated by econometric methods and then combined to form a synthesis of the whole. Tinbergen's model is an annual model of the United States (US), containing approximately fifty linear equations (the exact number of equations means little since it depends on the number of definitional relations adopted for convenience). The next relevant macromodel is Klein and Goldberger's [1955] model: again an annual model of the US, though condensed into about twenty equations, involving nonlinearities where necessary. It is perhaps this study which has firmly established the value of the method, stimulating further work on this and other models. The next work of relevance is the quarterly model of the British economy of Klein et al [1961], which has approximately thirty six equations dealing with the more complex nature of short run behaviour. Finally, perhaps the ultimate of this type of work to date, is the large (about a hundred and fifty equation) Brookings quarterly model of the US of Duesenberry et al [1965], which ties together narrower studies by a number of specialists. There are many more published macroeconometric models, each with their own characteristics, but these four present an adequate picture of the nature of the method. These models are mentioned because of their macro, exhaustive, nature; they are not discussed more fully since their properties have been made well known by a number of reviews and since they are of limited direct relevance to the building of sectoral models. Although they disaggregate by industry where appropriate, it is in such a manner that actual industry models are not inferred; disaggregation is essentially ad hoc: to increase accuracy through increasing homogeneity. The model presented in this study may be interpreted as a large macromodel, but this is not its main aim - it is essentially a model of intersectoral relations.

Sectoral models are discussed in some detail by Klein [1953], who divides them into two categories. The first of these is the 'general to particular' [p. 200] type of model, wherein a general economy model is constructed, on to which the particular sectoral model is grafted. This has the advantage of simplicity when dealing with one sector, but hides some of the important inter-industry relations; these points are not however relevant to exhaustive sectoral models. A seminal example of this approach is the Girshick and Haavelmo [1953] model of the demand for food. This contains five equations, of which the first expresses the demand for food in terms of, inter alia, current and previous income, and the third is a multiplier equation expressing income as a function of investment and previous income, being obtained directly from an aggregate consumption function of similar form to the demand for food equation, and the identity between income and consumption plus investment. These two are the most important equations, and show the 'general to particular' chain and the grafting of the food sector on to the economy model by explaining current income in the general model, which partially explains the demand for food in the particular. Other equations give the retail supply, production, and relative price of food.

Klein's second approach is general equilibrium analysis. The most relevant quantitative form of this is input-output analysis, stemming from Leontief's [1951] fixed coefficient model. This approach estimates the intermediary flows between industries in some base period, and thus a matrix of input-output coefficients, each element of which indicates the direct demands for the output of one industry made by some other industry per unit output of this (other) industry. Assumptions are then made about the behaviour of these coefficients, of which the simplest is that they are constant, and the resulting matrix, or some function of this, is then combined with a vector of final demands to give a prediction of required sectoral outputs. In its simplest form this model is primarily one of physical production rather than of economic behaviour, for the very nature of fixed coefficients precludes optimisation or choice.

A more complicated approach is the continuing work of the Cambridge Department of Applied Economics (CDAE), best summarised in CDAE [1964 July]. This is basically an input-output model, though submodels for an exhaustive set of sectors are formulated, which may be interpreted as subsystems whose main function is the determination of the input-output coefficients relating to that sector. These submodels are not however models in themselves in the sense that they may be separated from the whole and still remain meaningful.

Basically the CDAE model is a long term programming model, incorporating particular noneconomic information (in the shape of experts' opinions), and thus has important differences from a short term analytical model.

The work on models discussed above provides an important base for this study, which follows the overall methodology of econometric models, yet pursues a different approach to sectoral disaggregation. The model presented here attempts to bridge and build forward from both of Klein's categories of sectoral model: models which may stand by themselves are built for each of a set of mutually exclusive and exhaustive sectors, and are linked both horizontally (that is directly, through input-output relationships) and vertically (that is through aggregates connected by a skeletal economy model). Looking ahead it may be seen that the sectors investigated are industries, so to avoid confusion over various interpretations of the term 'sector' this is replaced from now on by 'industry'.

Profits There are many interpretations of the nature of profits; these may be divided into the orthodox, behavioural, and Marxian categories. There are also various definitions of profits, mainly differentiating between interest and 'pure profit', but also taking account of depreciation, rent, taxes and so forth. These interpretations and definitions are partly interdependent, but we will initially adopt the widest definition of profit, that is the gross reward of the capitalist; we may now briefly examine the interpretations.

The orthodox theory of profit states that profit is the reward for the accumulation of capital, or its supply price. The idea that capital requires a supply price has been severely criticised, not only by Marxist writers on the grounds that capitalists accumulate in order to maintain their class position, but also by (implicitly) existentialist views best summed up in Keynes' 'animal spirits'. We need not however enter this controversy, for this study is essentially concerned with profits in the short run, defined in the usual sense as the period when one factor, capital, is fixed. In the short run then the supply price of capital is not immediately relevant, for when capital stock is treated as given any sacrifice incurred in its accumulation is one of Jevons' byegones.

The behavioural theory of profit covers a wide range of theories of entrepreneurial behaviour, though its essence is that profits are the reward for the entrepreneurial factor, that is for the bearing of risk. This again assumes that there is some supply price of the relevant factor, that is that the entrepreneur is averted to risk. The economic, as opposed to psychological, rationale for risk aversion is based on the concept of diminishing marginal utility, and as the commitment of an entrepreneur in any one firm in a developed capitalist economy is frequently small (in terms of the expected income stream from the firm relative to his total income) this may not be important. The psychological motives for risk aversion must be taken as given in economic science, but observation would imply that such aversion is not great; further, if any individuals are neutral to risk these may be expected to be entrepreneurs. Perhaps the main defect of this interpretation appears when it is related to a developed capitalist economy with imperfect competition, for here entry into various industries is not free - its condition is the ownership of (or the ability to borrow) large amounts of capital. The level of profits is thus supported by the scarcity of entrepreneurs, which is determined not by the real costs of risk bearing but by the scarcity of individuals with anything to risk.

This leaves the Marxian theory of profit, that profit is pure surplus. This is the interpretation which is of most relevance to this study of profit as the main motivating force in the short run of an imperfectly competitive economy. Its motivational relevance is essentially that it is a pure surplus.

We may now return to the conceptual definition of profits - the details are discussed in chapter 2. As the motivational force of profits acts initially on those who control resources rather than their ultimate owners we are justified in defining profits to include interest as well as 'pure profit', for if interest is given in the short run it is the total reward which is relevant to the controller of capital. Profits should strictly be measured after allowing for depreciation, for this is clearly a disincentive to the use of capital; in the Marxian theory it is the only real cost of capital. No account of depreciation however is taken - for simplicity and for practical reasons discussed in chapters 2 and 3. Profits are measured before taxes, again for simplicity (though account may be taken of these separately). Finally, though there are various theories of the determination of rent both the orthodox and Marxian theories of rent treat it as a pure surplus as well (land being a 'free gift of nature'), and so rent is included in our definition of profits.

Thus profits are defined as the total reward of fixed factors, that is the excess of value added over the reward of labour. They are interpreted as a pure surplus, relevant as a motivating force; this is the reason for investigating profits.

Profit then is a surplus, or a residual, and as such cannot be validly examined by itself, but only in terms of the two flows of which it is the difference - total product and the reward of labour. This explains why there is very little analytical work on short run profit determination; there is however much work of a pragmatic, historical, nature on long run profits, which, though valuable in itself, is not of direct relevance here. There is one analytical short run study of particular interest, that of Evans [1968].

Evans' study develops a profits function at the industry level, where gross profits depend positively on sales and capacity utilisation, and negatively on past sales, and goes on to examine the inter-industry similarities and differences, and then (now on an aggregate level) investigates the relation between profits and labour costs. The second part of this paper is of considerable interest in illustrating the differences in competitive structures of industries, by examining the differing importances of the three 'determinants' of profit in each industry. In competitive industries high sales would mean more firms, and thus would have a modified effect on profits - though capacity utilisation would be relevant, and further, the negative effect of past sales would reflect the effect of new entrants to the industry; unit labour costs (included in trial formulations) would be correlated positively with profits. In more monopolistic industries sales would be expected to be more important, though not capacity utilisation, and past sales would be of less relevance (though they could indicate increased overhead costs due to past capital formation); unit labour costs would be negatively correlated with profits. The functions postulated are on the whole statistically significant, and the inferences about the competitive structures are in accordance with a priori notions and other measures. The third part of the study is, being at the aggregate level, of less relevance here: it combines an aggregate profits function with the five relevant wage and price determination equations of an aggregate economy model, and shows by simulating higher unit labour costs that these are reflected almost entirely in higher prices, decreasing profits very little. Evans' study constitutes a valuable investigation of competitive structures, but becomes of doubtful validity if the functions derived in the first part are treated as anything more than interesting empirical results, for example if they are interpreted as giving the 'determinants' of profits, for as profit is a residual it is not meaningful to make it the subject of a 'quasi-behavioural' relation if the usual interpretation of the nature of the firm is retained. Evans' ideas are however examined in a modified form in the analytical part of this study, for Evans' work (published while this study was well under way) provides virtually the only theoretical analysis of short term disaggregated profits. Indeed Evans [p. 343] commences: 'Although profits are often considered to be one of the most important variables in our economic system, very little empirical analysis has been undertaken to establish the determinants of corporate profits, and even less work has been done at the . . . industry level. This is somewhat surprising'.

The main relevance of profits has been made clear above: it is that they are the main motivating force in the economy, particularly as regards the allocation of resources among industries, which stresses the particular importance of industry profits.

Allied to this are the potential policy implications for a mixed enterprise economy. Most of the instruments of governmental control in such an economy are applied to profits in the sense that they are aimed at having their impact on (anticipated) profits and thence affecting the rest of the system indirectly, so that the main chain of control is instruments - profits - targets. An obvious example of this is action to affect investment, since investment may (in some theories) be considered as being directly determined by, inter alia, some form of profits. However the chain exists equally in less direct cases, prime examples in the UK being investment grants, selective employment taxes, and import duties, all of which have as one of their main aims the change of employment in certain industries, and are designed to achieve this through affecting profitability. Though the actual effectiveness of individual policies is a controversial matter, it is generally accepted that this mode of control is the essence of the mixed capitalist system.

Tatonnement General equilibrium theory of the first half of this century was content with inferring the existence of an equilibrium from the equality between the number of unknown variables in the system. It has thus missed the fundamental question of how the market solves these equations to arrive at the equilibrium solution; this is the question which may be answered by Walras' overlooked concept of tatonnement.

The theory of tatonnement states that the market itself solves the complex equilibrium equations by groping to the solution through successive approximations, a process which Walras [1954] explains in terms of some imaginery dialectical interaction between all the agents in the economy. This starts from some initial (disequilibrium) set of conditions (for example prices) for which all agents declare their intentions - intentions to produce, consume, and so forth. If all these intentions are not compatible, for example if aggregate demand does not equal aggregate supply, then the conditions are changed accordingly (in this example prices are increased if demand exceeds supply. This process is repeated until some set of conditions is found for which all intentions are compatible, whereupon these are consumated, and the next period is entered. Tatonnement thus describes a short run process, indicating the adjustment to the temporary equilibrium associated with each period, rather than the time path of these temporary equilibria over several periods. The relevant short time period here is the unit period of production and consumption planning - the Hicksian week. The process is thus dynamic but does not fall into the 'magnificent dynamics' of cycle and growth theory, though the latter may explain the intentions of each agent at each stage.

For this to be a complete theory of market adjustment it is necessary to assume recontracting and to explain the nature of the adjustment of the conditions, or parameters, at each stage in the process. Recontracting implies that agents withdraw their declared intentions at any stage in the process that produces disequilibrium, so there is no trading outside equilibrium; if this were not assumed then the resulting intermediary trading would affect the subsequent path of the tatonnement and thus the final equilibrium. It is this which distinguishes Walrasian tatonnement from the few alternative equilibriating processes where some intermediary trading is permitted. Walras was aware of the need for this assumption; he explains the declaration of intentions by the imaginery issue of tickets which are later scrapped. Walras is less explicit on the nature of the adjustment of the parameters at each stage in the process, indicating only the direction of the change. This is a possible defect in the Walrasian process, for the adequacy of the concept may depend on the form of this. In this sense the adequacy is interpreted as the ability of the process to converge to some stable temporary equilibrium, a point which Walras does not establish thoroughly but which has received much attention lately and is mentioned below.

The theory of tatonnement may be interpreted in the terminology of cybernetics as a specification of the 'black box' which produces an actual equilibrium from the system of mathematically determinate equations. The analysis of the process thus strengthens economic theory at its very centre. This does not necessarily imply a methodological error in accepting the black box approach for this may be an acceptable way of narrowing the scope of more particular investigations; the specification of this black box however may well have important implications for the understanding of the theory which uses it. This applies particularly in general equilibrium analysis, where the understanding of the black box is the essence of the problem. It is in this connection that Samuelson's [1947, chapter 9] observation that for an analysis of statics to yield fruitful results we must first develop a theory of dynamics is vital. As Koopmans [1957, essay 1] points out, there are many precedents in physics for bypassing questions of the mathematical existence of analytical constructs to obtain provisional properties of these constructs, but [p. 58] 'the fruits of such studies are like predated checks until the noncontradictory character of their premises has been established'. A similar position holds for empirical studies.

An alternative interpretation of the tatonnement process suggested by various critics is that it is merely a mathematical algorithm or pedagogic device, corresponding to the concept of the ether in physics. This view is incorrect: Walras [lesson 6] makes it quite clear that not only is tatonnement a practical process, but also that it is a particularly rapid and reliable one. The nature of the specification of a black box cannot however be conclusive, and it must be recognised that the behaviour of the market postulated by the tatonnement process is but one theory. It is however a highly plausible theory, and one which may be shown to be effective in producing an equilibrium solution of the system. It is thus (in the Hicksian sense) meaningful, for it would ideally be refutable if it could be shown to produce instable behaviour where a stable equilibrium were observable. Walras uses the concept of tatonnement to show how the market mechanism solves the equilibrium equations, not only of exchange [lesson 12], but also of production [lesson 21] and of 'capitalisation' [lesson 25]. Its original formulation is thus of general applicability to the whole area of economic analysis, and further, the essence of Walrasian tatonnement is profit motivation: 'this [tatonnement] is precisely the sort of groping which takes place spontaneously in the products market under conditions of free competition, as entrepreneurs increase or decrease their output according as they make profits or losses' [p. 247]. Developments of the theory per se all follow Walras closely, and are well documented by Patinkin [1965, note B]. This and more general work on tatonnement has concentrated exclusively on prices, these being considered as the only variables with a parametric function - explained by Lange [1936, p. 59] as 'the fact that, although the prices are a resultant of the behaviour of all individuals on the market, each individual separately regards the actual market prices as given data to which he has to adjust himself'. This study proposes a more general form of tatonnement where any appropriate variables may have a parametric function. The rationale of the parametric function of prices has been made clear, but this may also apply to quantities, where for example the output of firm A is (in part) a resultant of the behaviour of firm B, yet B regards this as given; this is most apparent in the theory of duopoly, but applies equally in the Leontief general equilibrium model. The importance of this extension rests on the possible difficulty of the price system alone in providing a determinate solution; it is thus particularly relevant to states of imperfect competition. Koopmans [1957, essay 1] has shown that under certain conditions a price system acting alone may not even solve the decision making problems of a single producer-consumer (Robinson Crusoe) economy, for [p. 32] 'communication in terms of quantities between the two Robinsons remains needed to avoid incompatible responses to the price system'.

This extension has one particularly important implication, for it makes the question of deciding how the conditions are to be altered from one stage in the process to the next redundant. This is because the parameters facing each agent at stage s in the process are determined explicitly by other agents in the economy at stage s-1; this is not the case with a simple price tatonnement where at no stage are the parameters (prices) determined explicitly by any agent. This property is illuminated in the discussion of the prescriptive applications of tatonnement later in this section and the nature of the overall model.

Previous work on tatonnement has been exclusively deductive, mainly because of the difficulties of observation (of the imaginery exchange of tickets before equilibrium is reached) necessary for an inductive approach. This study seeks to simulate the 'exchange of tickets' by the iterative solution of a multi-sector model, and examine the quantitative nature of the tatonnement process from the results of this.

We have suggested that the adequacy of the concept of tatonnement may be interpreted as the ability of the process to converge to some stable temporary equilibrium. Tattonement thus forms the basis of most of the work on the stability of a competitive economy, for alternatively the conditions for stability may be interpreted as the conditions for the applicability or convergence of the tatonnement process. This is first proposed in one of the seminal works in the field by Samuelson [1947, chapter 10], though despite numerous references to Walras he does not explicitly recognise tatonnement. This important field of economic theory is extensively summarised by Negishi [1962], and is not taken up here in its theoretical form. Negishi also examines the relevance to prescriptive economics of the analysis of stability, and this constitutes what is perhaps the most important application of the theory of tatonnement -

decentralised planning, or the efficient allocation of resources in a socialist economy. This approach originates with Lange [1936], and is followed by a number of later writers whose results are drawn together and extended by Malinvaud [1967]. As this is perhaps the most important application of the quantitative analysis of tatonnement and as it also illustrates the working of the process, the implications of decentralised planning based on tatonnement procedures are discussed below.

It is a basic result of welfare economics, shown for example by Samuelson [1947, chapter 8], that a competitive (interpreted in the strict, atomistic rather than the capitalist, laisser faire sense) equilibrium may be identified with a social optimum. The term 'optimum' is used in the Paretian sense to describe a state for which no feasible alternative state exists that would be chosen by at least one individual and avoided by none. As this concept is defined without assuming interpersonal utility comparisons it concerns allocative rather than distributive efficiency - or efficiency rather than equity. It is however possible to impose equity conditions on to a Pareto optimum if assumptions are made about individual 'sensitivities'. This is essentially an ethical matter, but if in default of value judgments the principle of Occam's razor is resorted to and diminishing marginal utility accepted, then this would indicate equality. As the strict competitive position clearly does not apply in a developed capitalist economy it may thus be desirable to simulate this position, such a simulation being provided by the tatonnement process. Alternatively the process may be interpreted as a device for achieving the efficient allocation of resources in a socialist economy, where for reasons of equity there is no market for some or all of these resources. The tatonnement process thus provides a solution to what is perhaps the fundamental problem of normative economics - how to combine equity with efficiency.

We specify the following model: there are m firms (or internally homogeneous industries) producing n goods, the net output of the i'th good by the j'th firm being x_{ij} and the consumption of this good being z_i, or in vector notation[2] $\underline{x}_j$ and $\underline{z}$ respectively. No stockpiling is assumed, so it follows that[3]

$$z_i \leqq S_j(x_{ij}), \qquad i = 1,\ldots,n,\ j = 1,\ldots,m,$$

the strict inequality implying free disposal. The system is also constrained so that the final outputs must be feasible (or acceptable) to the consumers, and so that the outputs of each industry should represent a transformation of inputs into outputs that is technically feasible, these feasible sets being given a priori. A plan H(k) for the economy is defined by the values that it gives to the $x_{ij}(k)$ and $z_i(k)$; it is a feasible if the final outputs and transformations mentioned above are both feasible. We assume at first, though this may be avoided, that there is some aggregate utility function $u = u(\underline{z}(k))$; a plan H(k) is then

2. Underlined lower case variables and parameters indicate vectors (and upper case matrices), the usual matrix notation applying; this should not involve confusion with (later) underlined industry identification symbols.

3. S( ) is the summation operator; $S_i(\ )$ is the summation operator over i.

preferable to a plan $H(k')$ if and only if $u(\underline{z}(k))$ exceeds $u(\underline{z}(k'))$. A plan is optimal if it is feasible and there is no preferable feasible plan.

The object of planning is to produce an optimal plan. This would be theoretically straightforward if the central planning agency were aware of all the relevant information, but this must be considered extremely unlikely in a complex developed economy; even if everything were known by the central agency the operation would be so complicated as to be virtually impossible in practice. It is fundamental to such procedures as we are considering that the central agency need not be omniscient, and that it need not do all the work - this is done in effect by the market itself. We assume then that the agency knows the feasible set of $\underline{z}$'s and the utility function $u(\underline{z})$ but not the feasible sets of $\underline{x}_j$'s, and that each firm j knows its own feasible set of technical possibilities $\underline{x}_j$ but not the sets for other firms or the feasible set of $\underline{z}$'s or the utility function $u(\underline{z})$.

The procedure is decentralised because of lack of knowledge, but this is circumvented by some dialectical learning process. This forms the essence of the plan: the agency devises a draft plan which it issues to the firms, and treating the information given in this draft as parameters the firms then submit their proposals to the agency. On the basis of these proposals the agency redrafts the plan, and the process is repeated as often as is desired. This procedure may clearly be a parallel to the equilibriating forces assumed in Walras' theory of tatonnement if certain conditions are satisfied. It is also partly a formalisation of the practices currently used in mixed economies, where the government will issue draft proposals to the representatives of various industries and other interested parties, invite their comments, and possibly modify the draft proposals (one or more times) in the light of these.

To define the procedure explicitly it is necessary to answer the following questions:

1 To which variables do the agency's drafts and the firms' proposals refer?

2 How are the firms proposals determined at each stage?

3 How are the agency's drafts determined at each stage, and how does the process begin?

4 How does the process end?

There are also a number of more practical problems concerned with the speed of communication and whether the firms act as required, but these are not considered here.

One form of decentralised procedure is then equivalent to Walras' tatonnement process, and in fact this is the most important form; as Malinvaud [1967, p. 180] mentions, 'Walrasian tatonnement seems to have been retained by all the authors who have given serious thought to [the] problem'. The basic tatonnement model is defined by the following answers to the four questions posed above.

1 The agency's drafts refer to the prices of the n commodities, $\underline{p}^s$, and the firms proposals refer to the outputs of these commodities, $\underline{x}_j^s$, each at stage s of the process.

2 At each stage each firm determines its proposal such that it is technically feasible and maximises its profits calculated at the draft prices $\underline{p}^s$; that is it maximises the

scalar product $\underline{p}^s\underline{x}_j$ over the feasible set of x_j.

3 At each stage the agency changes the draft prices in the direction of the excess demand indicated at the previous stage (having somehow determined the consumption demands at this stage), so

$$\underline{p}^s = \underline{p}^{s-1} + f(\underline{z}^{s-1} - S_j(\underline{x}_j^{s-1})),$$

where f is some increasing homogeneous function. The simplest form of such a rule is that the price of each commodity is changed by some constant positive proportion r of the excess demand for the commodity at the previous stage, so

$$\underline{p}^s = \underline{p}^{s-1} + r(\underline{z}^{s-1} - S_j(\underline{x}_j^{s-1}));$$

as this could result in negative values for some p_i these may be given lower bounds of zero.

4 The process may end when some optimal plan results (though it is difficult to see how this can be known), or more automatically when successive drafts coincide.

This is the basic tatonnement procedure; it is examined by Arrow and Hurwicz [1960] who investigate whether such a procedure will be effective, that is whether it will produce an optimal plan.

A major difficulty is that the plan might not be well defined, for two reasons. Firstly it might be impossible for firms to comply with their rule for determining their proposals at each stage given under (2) above, that is there may be no feasible vector $\underline{x}_j$ that maximises the firms profit $\underline{p}^s\underline{x}_j$ at stage s. This would occur if there were always increasing returns to scale so that profit increases indefinitely as output increases, that is if the feasible set of $\underline{x}_j$ were not bounded; it could also occur if this set were not closed, though we may more reasonably assume this - so that the limit vector of a convergent sequence of feasible vectors $\underline{x}_j$ is also feasible. A second reason why the plan might not be well defined is that there may be no feasible consumption vector $\underline{z}$, that is the feasible set of $\underline{z}$ may contain no vector $\underline{z}$ all components of which do not exceed the corresponding components of the vector $S_j(\underline{x}_j)$. To ensure that the plan is well defined we thus assume that the feasible sets of $\underline{x}_j$ and of $\underline{z}$ are closed and convex, and that the function $u(\underline{z})$ is continuous and concave. These assumptions (which are slightly stronger than are immediately needed) imply nonincreasing returns to scale and decreasing marginal utilities of all commodities.

Having ensured that the plan is well defined it is now necessary to show that will be optimal. This is done by Arrow and Hurwicz in a way which is similar to the demonstration of the stability of a competitive economy; it is shown that for an indefinitely increasing number of stages a price vector will result which is associated with an optimal programme. This result is theoretically important, but the rather restrictive assumptions which are required to ensure convergence make its practical use more doubtful; it also suggests that there may be more theoretically efficient procedures.

A possible alternative procedure is implied by this study, for the more general tatonnement adjustment process investigated could form the basis of such a planning procedure.

In terms of answers to the four questions posed above this would be defined as:

1 The agency's drafts refer to all the external variables that the firm considers relevant to the determination of its behaviour. This is a more natural process, for rather restrictive assumptions must be made if prices are to be considered as the only relevant external variable in the firm's decision making process; it may also be expected to be more effective, for the use of additional (relevant) information should make the decision making process more efficient.

2 The firms proposals at each stage are determined in the same way as in the basic tatonnement procedure; that is each firm acts so as to maximise its profit given the parameters implied by the agency's latest draft.

3 The agency's drafts are determined automatically from the firms' proposals, in that the agency's drafts at stage s are related by identities to the firms' proposals at stage s-1; the actual adjustment is thus determined (implicitly) by the knowledgeable firms rather than by the unknowledgeable agency. It is assumed for simplicity that the agency processes the replies of firms; a theoretically equivalent procedure is obtained by the agency merely issuing to each firm as its draft at stage s the proposals of all firms at stage s-1 and letting each firm process this information - though in practice this might encourage firms to act improperly relative to (2) above. The agency's draft at stage s must also of course depend on the proposals of consumers at stage s-1 given the parameters implied by the

draft at stags s-1. This may be achieved conceptually by treating all consumers as a class of firm acting identically to all other firms in the process other than that they act so as to maximise their utilities rather than their profits at each stage. In practice this would require a smaller number of representative consumers, or a model of consumer behaviour operated by the agency. An example of the specific identities connecting the agency's draft at stage s and the firms proposals at stage s-1 is provided by the skeletal economy model developed in chapter 3.

4 The process ends automatically when the vector of the variables issued as the agency's draft at stage s-1 converges to this vector at stage s; in practice this would be interpreted as occurring when some norm of the vectors at two consecutive stages differed by some arbitrary small (proportional) amount. Clearly when this occurs the system is in equilibrium, for if firms act consistently then they will return the same proposals at stage s as at stage s-1, resulting in the same drafts from the agency at stage s+1, and so forth. Assuming that the firms act properly, as interpreted in (2) above, their behaviour is then equivalent to perfectly competitive behaviour in that profits are maximised on the assumption that all variables external to the firm are given and thus cannot be influenced by the firm. The identification of this plan with an optimal plan then follows automatically from the identification of the competitive equilibrium with the social optimum.

Such a procedure has three important advantages over the more basic tatonnement procedure. Firstly, the operations of the agency are purely routine and require no judgement, thus leaving to firms the entire decision making process - an activity which constitutes their raison d'etre. Secondly, or possibly a particular case of this, the agency need not know the social utility function, the existence of which is not even required; indeed no value judgements are required save that ceteris paribus more output of any commodity (and similarly less input for the same output) is desirable. Thirdly the procedure is more general in that it encompasses all the relevant variables external to the firm but endogenous to the economy. The advantage of this is that although prices may often be the only relevant parameters under conditions of perfect competition this is not generally the case under conditions of imperfect competition, with which we are necessarily concerned; this imperfect structure may mean that prices do not manifest themselves throughout the productive process. It is then desirable to simulate the entire competitive environment, not just prices. This is implicit in Samuelson's [1947, p. 232] demonstration that a planning agency using a basic tatonnement process would avoid the minimisation of prices although this may be exactly what is required for a social optimum under conditions of decreasing cost. Samuelson then concludes that 'the decentralized operators in a planned society should refrain from a literal aping of atomistic, passive, parametric price behaviour. Instead of pretending that demand curves are infinitely elastic when they are not, the correct shape of the curve is to be taken into account'. The need for taking account of the whole environment is similar for that of the shape of the demand curve.

Such a generalised tatonnement procedure may then be valuable. Its greater scope however makes its theoretical properties rather more intractable, and these are not investigated. Instead of this one of the aims of this study is to examine the quantitative properties of such a process by using econometric models to simulate the behaviour of all the principals in the process.

1-3 The model

The purpose and background of this study have been discussed above; this section provides a conceptual introduction to the core of the study, the model: its aims, the basic method, and its essence - the linkages.

Aims It is relevant at this stage to clarify the purpose of the model (as opposed to the whole study). This is fundamentally the understanding of economic forces and their resolution for, in Marshall's phrase, 'fruit or light'; it is not (directly, though possibly indirectly) forecasting or planning. Accordingly the model is confined to meaningful and measurable economic variables, and excludes ad hoc exogenous variables which may increase predictive accuracy but explain little. Prediction may be the acid test of even an explanatory model (and indeed the need for accuracy is stressed), but predictive power is always relative to the amount of external information used.

Method The overall model comprises separate submodels for a number of mutually exclusive and exhaustive industries, each of which incorporates any links with other submodels or with the aggregate that are appropriate, yet may be thought of as self contained for given values of these link variables. These submodels are connected by a skeletal aggregate model.

The method has three main stages. Firstly a general outline is built up, detailing the type of underlying structure of decision making rules, technical and institutional relations and so forth that are expected to be relevant to any industry. Secondly this general industry outline is examined from both a theoretical and an empirical standpoint, and modified to apply to each of the industries under consideration. Finally, the aggregate economy model is built, into which the various industry models dovetail; this may include any economic relations not properly assignable to any one industry, as well as any desired consistency criteria. If the outline of the first stage is properly specified it will need little modification to apply to the various industries, for the sort of relationship it embodies are the sort that are expected to apply whatever the nature of the industry - in terms of the technology employed, or of the state of competition. For example, if it is suggested that the decision to add to capital stock depends partly on the level of capital stock (for replacement), partly on the change in output, and partly on past profits (or liquidity), this would presumably apply to any industry. It may well be that the three factors have differing degrees of importance (including zero importance) in different industries, but this will merely be reflected in the different values of the coefficients. In the investment example an industry with a new or particularly durable type of capital stock (for example the real estate industry) may perform virtually no replacement investment; a fashionable growth industry with ready access to the capital markets may not be concerned with liquidity. Thus an ideal outline model should contain most of the variables likely to influence relationships in any industry, so that the second stage consists mainly of removing from each industry those factors which are not relevant, and only to a lesser extent the addition of variables particularly relevant to the industry. The outline model itself should however be reasonably interpretable, if not workable, and so should remain free from variables obviously peculiar to only one industry. Thus the outline model may be of some direct value, and the comparison of modifications to it in various industries may illustrate relevant differences in their natures.

This approach may alternatively be interpreted as a type of mixed cross section and time series study, for it requires that relationships must accord with empirical observation over industries as well as over time. A similar approach could specify a cross section relationship that should hold over time (as opposed to a time series relationship that should hold over industries), but this would be less useful for further development.

Linkages As the linkages between the submodels are fundamental to the study it is appropriate to discuss their general nature at this stage. These linkages fall into two categories: the horizontal links, or direct relations between one industry and another, and the vertical links, or relations between one industry and the whole. The first of these categories covers the case where a variable in one industry depends on the corresponding variable in other industries, usually on a weighted average of these, the weights being derived from an input-output matrix. For example the production decision in one industry may depend on the anticipated demand, which may depend in part on the requirements for the product of this industry by other industries, that is the sum of the outputs of these industries weighted by the relevant input-output coefficients; alternatively the average price of materials purchased by an industry will depend on (ideally be identical to) a weighted average of the prices charged by the industries that supply it, the weights again being derived from the relevant input-output coefficients. This form of linkage illustrates the connection with the general equilibrium approach; the basic difference is that the predominantly technical information supplied by the input-output matrix is used in predominantly mechanical relations where it is valuable, whilst more flexible behavioural relations are treated by more appropriate flexible methods.

The second type of linkage covers the case where an economy variable, either absolutely or in ratio with the corresponding industry variable, is important in determining part of an industry's behaviour, this economy variable being the aggregate of the corresponding variables in all industries. For example the final demand for the product of an industry may depend on the aggregate level of income (the sum of the values added by this and all other industries), and the price of this product relative to overall prices (the weighted - by product - average of the prices charged by this and all other industries). This form of linkage illustrates the connection with the 'general to particular' approach; the basic difference is that there is an exhaustive set of submodels so that the aggregate is given by all the submodels together, not formulated explicitly: it thus becomes 'particular to general' as well.

The above remarks describe the conceptual nature of the theoretical model which is at the heart of this study. It must however be noted that the study seeks to provide a tested methodology, of which the theoretical model is but the basis. This involves the collection of data, estimation, and solution of the model, and these parts taken as a whole form the thesis. The aim of this section is to present the basis of the study rather than a summary; thus only the conceptual nature of the theoretical model is introduced at this stage.

1-4 Outline

This chapter, 1, has introduced the study and the model. As this is based on the concurrent development of theory and observation, it is necessary at an early stage to make observations available: in this case to specify, collect, and adjust the required data series. This is done in chapter 2, which chooses the area of reality to be observed, and in what detail it is to be observed, as well as collecting and collating the individual series, which are presented in appendix B. For a short term disaggregated model this presents a number of both conceptual and practical problems.

With this available, chapter 3 develops the theoretical model by a combination of deduction from a priori knowledge and induction from observation. This is the model introduced in the preceeding section - the basis of the study.

The main area of econometric theory is that of estimation of parameters, and this is crucial to the study, for the stochastic nature of the model implies that different methods of estimation will produce different results. Thus the theory of estimation is discussed in chapter 4, and a method is adopted. This chapter then discusses the results and refinements which arise from applying the method to the theoretical model; the estimates themselves are given in appendix D.

The aim of the study is not only to present a model, but also to show how it may be tested and indicate the nature of the results which such a method might be expected to produce; this is the subject of chapter 5. The model is large and reasonably complex, and thus its solution is complicated. The problem is discussed, and a method is proposed; the model is then solved by this method, and tested in a number of ways, the solutions being presented in appendix E.

Finally, chapter 6 brings together the main results on the adequacy of the model, and draws some conclusions from these.

References appear in appendix A, and two papers on computation are given in appendix C.

CHAPTER 2 THE DATA

The model, due to its disaggregation, requires a considerable amount of data. The collection and revision of this comprises a significant proportion of the study, and the resulting consistent set of figures is a possibly important by-product. For this reason the data used is given in full in appendix B; as figures are of little use without a reasonably detailed account of their derivation, and because in certain areas the method chosen for revision of the raw data is partly subjective, the definitions and derivation of the final figures are also given in full, in this chapter.

2-1 Area

Geographical The model concerns an economy, and so for institutional reasons must be restricted to one country, or at most one free trade area. In its suggested form the model would only be valid for a developed, capitalist economy, though if data were available there is no reason why similar models should not apply to a socialist or to a developing economy. Apart from this restriction the choice of country is open, and the decision to examine the UK is determined by the relative ease of access to data; it is hoped that the basic structure, though not the values of the parameters, would apply to any developed capitalist economy.

Temporal The main observations on the choice of economy also apply to the choice of time period - used here to cover both the sample period, over which the parameters are estimated, and prediction period, over which the model is tested; these two are mutually exclusive and exhaust the time period. It has been argued by Christ [1966, p. 547] for example that 'there is nothing that can be learned by saving some data for testing predictions that cannot also be learned by estimating on the basis of all the available data and examining all the residuals'; thus prediction only fills its role of the acid test of a model where it concerns a completely new set of data, with which the investigator is unfamiliar when the model is chosen. This represents an ideal, for it may be argued that the investigator cannot completely isolate himself from history while it is occurring, and thus implicitly uses information from the prediction period in formulating the model; this is allied to Locke's arguments on the impossibility of a priori knowledge - that this is implicit empirical knowledge. In practice the more usual approach is perhaps acceptable (and is in fact later adopted by Christ in the work cited), for if accuracy of prediction is to be assessed it is necessary either to save some of the immediately available data for prediction or to wait until more becomes available. The latter course is considered impractical here due to the cost of the delay, mainly in terms of the diseconomies of revising data in two groups. Thus part of the available data is reserved for prediction, and apart from being examined to determine the best method of making all the data compatible, is ignored until the estimated model is tested. Reserving the earlier part of the time period might have been preferable in that it would have meant that the estimated model was based on the most recent data, but, for a few variables, isolated observations preceeding the main sample period are needed for initial values where those variables are lagged, so the latter part is reserved for prediction.

The obvious choice for the end of the time period is the most recent reliable observation (observations of macro-economic variables tend to be partly estimates, and very recent observations tend to be unreliable estimates - witness their sometimes major revision); this is accepted. The length of the time period is obtained by balancing the extra usefulness, in terms of degrees of freedom, against the cost, in terms of time and possible unreliability of obtaining distant figures - a long time period also increases the probability of a change in the structure. The choice of the length of the period then depends on the frequency of the observations, which (as is decided below) is quarterly. Thus the fourth quarter of the year 1966 (1966 Q4) is given as the end of the time period, and if four quarters are saved for prediction, the end of the sample period is given as 1965 Q4. Increasing difficulty in obtaining data before 1958 was encountered, and so 1956 Q1 is taken as the start of the sample period, making this a decade, or forty observations. Only four quarters are allocated to prediction on the grounds that it is more fruitful to test rather cursorily a model built from a sound base than to investigate more thoroughly a model with weaker foundations; four observations, or one year, may be taken as a (absolute) minimum requirement for valid assessment.

Frequency It has been made clear that this is a study of short run profits and adjustment. One of the main reasons for this is that the tatonnement process is concerned with temporary equilibrium, and so a short, probably the shortest practical, time period is appropriate. This is taken as a quarter of a year: annual figures are usually readily available, but a year is too long, while monthly figures, though preferable, are not available for many variables. As it is, compilation of consistent disaggregated quarterly figures poses many problems.

Perhaps the most important general problem concerns seasonality, which is discussed here, although, as it transpires, this may be considered part of the model. In fact no attempt is made to adjust any series for seasonal variation, since the choice of the method of seasonal adjustment must be to some extent arbitrary, it being considered more useful to give the original figures and make the adjustment process explicit. It is well known (and shown for example by Klein [1953, chapter 7]) that the usual nonparametric methods of seasonal adjustment (mainly comparison with seasonally similar periods and moving averages) suffer from serious statistical deficiencies - most notably, though not only, autocorrelation. Thus unadjusted series are used throughout, and allowance for seasonal factors is made through the introduction of specific seasonal variables in each equation, as suggested by Klein. This is in effect the introduction of three dummy variables, QU, QD, QT, which have a value of unity in the first (unum), second (duo), and third (tres) quarters respectively, otherwise being zero. Implicit in this is a fourth dummy which is always zero, which achieves the convention of treating the fourth quarter's value as the true value. A typical equation may thus be written in the form

$$y_t = \alpha_0 + \alpha_1 QU_t + \alpha_2 QD_t + \alpha_3 QT_t + \beta x_t + u_t,$$

where y_t, x_t, and u_t are respectively the values of the dependent variable, independent variable(s), and residual in period t, and the α's and β are parameters. This approach has the advantage of showing the depletion of the number of degrees of freedom (three, which may be afforded in a sample of forty observations), and allowing the valid use of normal methods of statistical inference. It does however depend on the assumption that the seasonal effects are additive and linear; this could be overcome at the cost of greater complexity, but the simple hypothesis is retained because in the present state of econometrics it is a generally accepted approximation that the influence of various factors (which may be nonlinear combinations of variables) is additive and linear, and because Klein et al [1961, p. 43] have shown empirically in a comparable field that 'the seasonal scatter of points conforms clearly to a pattern of parallel layers of relationships. The slope coefficients appear to have no significant seasonal variation'; this assumption is not tested again here. It should be noted that this method does not directly produce information on the seasonal variation of each variable, but on the variation in each equation, where it is an aggregate of the variations of each variable in the equation. Seasonal adjustment of variables themselves could be achieved by estimating the parameters of the equation

$$y_t = \alpha_0 + \alpha_1 QU_t + \alpha_2 QD_t + \alpha_3 QT_t + u_t$$

but this is not relevant here.

2-2 Framework

Sectoral disaggregation is fundamental to the model. There are many interpretations of 'sector' - type of industry, type of commodity, form of ownership, geographical area; as mentioned in chapter 1 this study uses the first of these, industry (which, as is discussed below, is taken as being equivalent to commodity). The term 'industry' is used in its widest sense to include all classifications of economic activity; indeed for terminological simplicity even governmental activity is referred to as an industry, though no model appears for it.

After subtracting government the first disaggregation is the division of activity into manufacturing and nonmanufacturing. A basic decision is now taken as regards nonmanufacturing: to treat it as one industry. This is obviously undesirable because of its clear heterogeneity and relatively large size (see table 2-1), but is made inevitable by its paucity of disaggregated quarterly data. This then cannot be avoided; it may be interpreted as defining more explicitly the scope of the model: a model of manufacturing industries with nonmanufacturing as a residual. Within manufacturing the degree of disaggregation is obtained by balancing the benefits of the homogeneity implicit in a large number of small groups against the increased noise, difficulty of obtaining data, and time spent in constructing a large number of industry models.

The basis for disaggregation is the Standard Industrial Classification (SIC) for 1958, defined in CSO [1958], since most published data is based on this, with the SIC order as the minimum size for a group. A further criterion is that groups within manufacturing should be of similar size. One of the simplest interpretations of this is the weight in the index of industrial production, given by CSO [1959], and accordingly all orders with such a weight of less than 50 are aggregated with the most appropriate other order, which reduces the number of groups from fourteen to nine. All of these resulting groups appear reasonably homogeneous, with the exception of one containing orders VII (shipbuilding and marine engineering) and IX (metal goods not elsewhere specified), and one containing orders XIII (bricks, pottery, glass, cement, et cetera), XIV (timber, furniture, et cetera), and XVI (other manufacturing industries). The former is retained as together with orders VI (engineering and electrical goods) and VIII (vehicles) it comprises a larger SIC grouping (engineering and allied industries) for which only aggregate data is frequently given; the latter is a necessary residual which merely enlarges the SIC residual. This provides ten basic industries, an acceptable number; their detailed breakdown and a comparison with other classifications is given in table 2-1. This also provides a symbolic identification for the industries, by which they will be referred to in future (these symbols always appear underlined; <u>mfg</u> refers to any or all of the categories in <u>m</u>). These are repeated in the glossary in appendix B.

It is apparent that any two of the industries are superfluous; in fact figures for one industry have frequently been derived from others. It is however appropriate to give figures for all industries for in some cases (where different sources have been used or where one source uses different methods) figures for the aggregates (<u>s</u> and <u>m</u>) differ from the sum of their components, and recording both makes any statistical discrepancy explicit.

2-3 Conventions

All variables used in the study are recorded, though the revision of the model during its construction has made some of these redundant; they are given for as many industry groups as is possible and meaningful.

As has been mentioned no series is seasonally adjusted. For a similar reason, that adjustment should wherever possible be explicit, all value series are measured at current prices, rather than using published constant price series, involving possibly arbitrary price deflators.

Abbreviations are used for variables and industries where appropriate; symbols representing variables are given in the subheading introducing them; they are also reproduced in the glossary in appendix B.

This section avoids repetition when discussing the individual series; there are however a few exceptions to the

Table 2-1 The industry breakdown

Symbol	Industry	Weight[a]	Order SIC 1958[b]	Order SIC 1948[c]	The Economist grouping[d]	CDAE cat.y[e]	SITC division[f]
s	sum total	(1902)[g]	I - XXIII	I-XX, XXIII, XXIV	total: all gps	1 - 31	0 - 9
g	(government)	-	XXIV	XXI, XXII	-	-	-
m	manufacturing	748	III - XVI	III - XVI	(all mfg)	4 - 24	5 - 8
n	non-manufact.g	(1154)[g]	I, II, XVII - XXIII	I, II, XVII, XX, XXIII, XXIV	(s less m)	1 - 3, 25 - 31	0 - 4, 9
f	food, drink, tobacco	86	III	XIII	breweries etc; food mfg; tob.o	4, 5	-
c	chemicals	68	IV	IV	chemicals etc	6 - 8	5
i	iron, steel, met	68	V	V	iron and steel	9 - 11	67 - 69
e	engineering goods	167	VI	with a: VI, VIII, IX	electrical etc. engineering	12	)
v	vehicles	79	VIII	VII	motors etc	14 - 16	) 7
a	allied engineering	64	VII, IX	with e: VI, VIII, IX	shipbuilding; household etc	13, 17	)
t	textiles, clothing, footwear	92	X - XII	X - XII	testiles; clothing etc	18, 19	65
p	paper, printing publishing	55	XV	XV	paper etc; publishing etc	23	)61 - 64,
o	other manuf.g	70	XIII, XIV, XVI	III, XIV, XVI	building etc; miscellaneous	20, 21 22, 24	)66,) 8

a CSO [1959]; b CSO [1958]; c CSO [1948]; d The Economist: Industrial profits and assets, April - June 1967; e CDAE [1963]; f UN [1961]; g see section 2-7.

general conventions, and these are mentioned in context.

Units As the model is based on an accounting framework rather than on index numbers the units are important. These are listed below:

1 Money flow variables are in £ millions per quarter.

2 Stock variables are in £ millions or thousands of persons, as appropriate, at the end of the quarter.

3 Rates (such as tax rates or interest rates) are kept as proportions per annum.

4 Indices (such as those of production or prices) are based so that the average over the sample period (1956-1965) is unity. All figures are given to the maximum number of places that is meaningful assuming that the sources do likewise. No mention is made of the conversion from monthly source figures to quarterly figures unless this is of particular interest.

Sources Published sources are used predominantly, and in the few cases where unpublished material has to be used the original figures from this material are given in full in the text; thus duplication of all the series in appendix B is possible. The main published sources are the Monthly Digest of Statistics (MDS) and National Income and Expenditure (NIE); frequent reference is made to these in their abbreviated form followed by a number referring to the relevant table number in the most recent copy used: that is the December 1967 MDS and the 1967 NIE. (This is perhaps rather early, but logistically necessary.) Within this limit in all cases where these and other periodicals are used the most recently published figures are taken. Further details of published series are frequently given in the table from which they originate or in the text, appendix, or supplement of the source; in general no such references are made, but should be inferred. The overall methodology and particular details of all national income statistics are given in CSO [1968], and this again is not in general referred to explicitly.

2-4 Industry series

Output (X) Figures are quantum indices of gross output, discussed in detail in CSO [1959], as they are the only readily available figures. Possibly superior approximations to indices of net output may be derived from the changes in gross output weighted in terms of some base period's net output, but this is not attempted here. Gross output is relevant to production decision equations, but can only act as a proxy for net output, or value added, in production functions. The basic figures are from MDS 46, aggregated where necessary at the published weights. The published figures up to 1958 Q3 are based on 1954 and since then on 1958; the two series are thus spliced by regressing the old series on the new for the available overlap period (ten quarters). This applies only to industries m and mfg; figures for g are obtained by interpolating annual figures of gross national product attributable to government (that is public administration and defence, and public health and educational services - from NIE 11, deflated by the deflator of gross domestic product implicit in MDS 1) according to quarterly figures for government expenditure on labour (see below). Gross domestic product (see below) is used for s+g, the other aggregates being obtained from these by subtraction - m being weighted by the value of its contribution to gross national product in the base year 1958, from NIE 11.

Labour (L) These are numbers of male and female, part time and full time, employees in Great Britain (GB), that is UK less Northern Ireland - as figures for the UK are not readily available; they apply to the middle of the quarter and are taken from MDS 14. Figures for s are those for the total in civil employment, less national and local government service (g). Various changes occur during the time period. In 1959 Q2 the basis changes from SIC 1948 to SIC 1958; this is allowed for by adjusting the earlier figures by the ratio of the figures of the two classifications for this quarter, no longer overlap being available. Further, under the SIC 1948 industries e and a are aggregated; these are separated according to figures from the Ministry of Labour Gazette, the source of the MDS figures. The published figures are again revised in 1964 Q3, earlier observations being adjusted according to the ratio in that quarter. The fact that all figures before 1962 Q3 are taken two thirds of the way through the quarter is ignored. The last year of the time period again involves a change, and figures for this year are taken from the Ministry of Labour Gazette, table 103 (m and mfg) and Statistics on Incomes, Prices, Employment and Production, table E 1 (s and g). Throughout figures for n are obtained by subtraction.

It should perhaps be emphasised that the term 'labour' is used throughout to refer to those actually employed - as opposed to the 'labour force' which includes those unemployed.

Unemployment (U) Figures for unemployment are similar to those for labour, and again apply to GB; they originate from the Ministry of Labour Gazette 'Industrial Analysis of Unemployment' table; they are the total of males and females wholly and temporarily unemployed. In this case figures for the UK are available but are discarded in favour of those for GB to maintain consistency with the labour figures. The change from SIC 1948 to SIC 1958 in 1959 Q3 is treated analagously to the change for labour, using the labour ratio as none is available for unemployment. Again figures for industries m (until 1960 Q1) and n are obtained from the others.

Investment (I) Figures are of gross fixed investment in all types of asset, and are taken almost directly from MDS 8. Industries e and a are combined in the source and are separated according to their relative proportions in each year (from NIE 60). Figures for industries s and g are from MDS 7, the public corporation element of the figure for g before 1964 Q1 being deducted according to the relative proportions of public corporations and public authorities in the total (from NIE 54) - this gives, as a check, reasonable figures for the period since 1964 Q1. Industry n is again a residual.

It may be noted however that MDS 8 gives extremely high figures for the year 1956, the total for which exceeds the corresponding figure given by NIE 60 by 74 percent. It is usual to expect some error in quarterly figures, especially for relatively volatile series such as investment, but this appears excessive. Accordingly all figures for 1956 are divided by this factor (1·74) after allowing for the 'normal discrepancy' - taken as that for the year 1957.

<u>Stocks (S)</u> These are total inventory stocks (materials and fuel, work in progress, and finished goods), the published figures for which are discussed in <u>Economic Trends</u> August 1960. The series are derived from figures for a base date and changes, sometimes as the value of physical increase and sometimes as the increase in book value, for preceeding and subsequent quarters. Figures for the period up to 1959 Q4 are obtained by subtracting cumulatively the change in value of stocks at current prices for each quarter from the value of stocks held at the end of 1959 Q4; all figures are for MDS 6 except those for industry <u>s</u> (from MDS 2), the figures for <u>i</u> plus <u>e+v+a</u> and for <u>c</u> plus <u>p+o</u> being separated according to their ratios at the end of 1959 Q4. After 1959 Q4 the process is rather more complex: the values of the physical increase in each quarter at current prices are deflated by the relevant price indices (see below) to give figures for the values of physical increase at constant (1958) prices (though in fact these are given directly at 1958 prices until 1962 Q4) which are added cumulatively to the value of stocks held at 1958 prices at the end of 1959 Q4. The

resulting figures of stocks held at 1958 prices are then reflated by the same price indices to give stocks held at current prices. This gives figures for industries s, m, and mfg (though no attempt is made to separate industries e+v+a and p+o). Figures for g for the whole period are obtained by subtracting cumulatively the increase in book value of stocks by year from their value at the end of 1966, both at current prices (from NIE 71), to give figures for the fourth quarters, and linearly interpolating to give figures for the first, second, and third quarters in each year. Figures for industry n are then obtained by subtraction.

The price indices used are weighted averages of the input and output prices (see below) for each industry, the weights being: for input prices the value of materials and fuel held plus (an arbitrary) half of the value of work in progress at the end of 1959 Q4, and for output prices the value of finished products plus half of the value of work in progress at the same date - both expressed as proportions of the total; for industry s however, arbitrary weights of a half each are taken due to the lack of data. Ideally the three types of inventory asset should be treated separately as should the three types of fixed asset (see below), but this is not attempted. Where there is no exactly corresponding wholesale price the assumed next best is used; these series, together with the weights, are given in table 2-2 (industries n and g do not appear since their series are not adjusted by price indices).

Table 2-2 Price deflators for stocks

Industry	Series used: Input price	Series used: Output price	Weights: Input price	Weights: Output price
s	m	m	0·500	0·500
m	m	m	0·532	0·468
f	f	f	0·580	0·420
c	c	c	0·516	0·484
i	e	i	0·607	0·393
e+v+a	e	i	0·526	0·474
t	t	t	0·476	0·524
p+o	p	p	0·510	0·490

Price (P) Figures are indices of the ex works prices of output, and exclude any purchase taxes - though they include excise duty; they cover industries m, f, c, i, t, p, and a residual e+v+a+o and are from MDS 167. The series for industry m, which covers home market sales only, is given in a revised form from 1963 Q1, so the earlier part of the series is adjusted according to a regression of the new series on the old over the overlap period of eight quarters. Figures for the residual industry and industry t (from 'textile industries other than clothing' and 'clothing and footwear') are weighted averages, the weights being those of the index of production. As no quarterly figures are available for 1956 the annual figure is taken for all quarters; other methods of allocation were considered, particularly the allocation of the year's figure so that a linear trend is maintained over the first five quarters, but being more complex and not necessarily more appropriate, were not used.

<u>Earnings (E)</u> These are average quarterly earnings of all employees, male and female, wage and salary receiving, ..essed in £ thousands, so that earnings multiplied by labour produces labour income in £ millions. The figures are derived in three main stages. Firstly, figures of average weekly earnings for adult men manual workers in the second pay week of April and October each year (as published in the <u>Ministry of Labour Gazette</u> statistical series, table 122 (December 1967 issue)) are assigned to the second and fourth quarters of each year. Until 1959 Q4 these figures are based on the 1948 SIC and industries <u>e</u> and <u>a</u> are combined; allowance is made for the change from the 1948 to the 1958 SIC by adjusting the earlier series by the ratio of the figures for the two classifications for 1959 Q4, and industries <u>e</u> and <u>a</u> are separated according to their (almost equal) means over the rest of the period. Secondly, figures of average earnings of all employees for all quarters from 1963 Q1 are taken from the <u>Ministry of Labour Gazette</u> statistical series, table 127 (December 1967 issue), and the figures from the first stage are regressed on these for each second and fourth quarter from 1963 Q2 onwards and are adjusted accordingly. Figures for the first and third quarters before 1963 Q1 are then derived by interpolating the second and fourth quarter figures according to the mean seasonal from 1963 Q1 on, so

$$e_t^3 = e_t^2 + (e_t^4 - e_t^2).S_t(e_t^3 - e_t^2)/S_t(e_t^4 - e_t^2),$$

and

$$e_t^1 = e_{t-1}^4 + (e_t^2 - e_{t-1}^4).S_t(e_t^1 - e_{t-1}^4)/S_t(e_t^2 - e_{t-1}^4),$$

where e_t^q is earnings in quarter q of year t, and the summation operator is over the years 1963 to 1966. This process effectively maintains the same seasonal throughout the series, rather than introducing possible distortions by incorporating a different seasonal in the first part of the series - where the difference is entirely attributable to the interpolating process. Thirdly, the series from the second stage are used to interpolate annual figures of total wages and salaries (from NIE 18 for m and mfg, NIE 17 for s) divided by the number of employees (same source for m and mfg, see labour series for s) to give average quarterly earnings. Finally, figures for n are obtained as the weighted (by labour) difference between s and m.

Three points emerge from this. Firstly, the desirability of dividing aggregate earnings into (at least) wage earnings and salary earnings becomes apparent; it can be countered by the need for simplicity in a model aiming at disaggregation by industry rather than by variable, and by the closeness of the two series. This is illustrated graphically in the Ministry of Labour Gazette (December 1967), which shows salaries (earnings or rates - presumably equivalent), weekly wage earnings, and weekly wage rates over the period: there are marked differences between salaries and wage rates, but salaries and wage earnings follow an almost identical path, which suggests that it may be acceptable to combine the two. Secondly, further insight may be obtainable at the cost of greater complexity by dividing earnings in another dimension, into hourly earnings and average hours; this again is omitted mainly for simplicity - the point is further discussed in chapter 3. Thirdly, the final series have been obtained in a rather circumspect way, which includes the splicing of series for men manual workers with those for all workers. The resulting series is however only used to interpolate an annual series, and in fact the degree of correlation between the two spliced series is high: in only one industry (v, where it is 0·90) is the squared correlation coefficient (for eight observations) less than 0·95.

Profits (Z) These are gross trading profits of companies operating in the UK; they exclude (or are measured before provision for) depreciation, stock appreciation, tax, interest payments, investment and rental income, and net income derived from abroad - that is they are the reward of fixed factors for domestic production. The basic figures are those published by *The Economist* of trading profits of companies (disaggregated into twenty three industries) reporting in each quarter, being given both for the latest year and for the previous year for the same companies. These are combined into a continuous series by a method suggested by Prais [1957], which comprises three main stages. Firstly, a lag of two quarters is introduced to allow rather arbitrarily for the delay between the earning and the reporting of profits. Secondly, the figures are partially aggregated to conform to our classification of industries, and for each industry for each quarter linked series are derived showing the year to year movements in profits for the companies reporting in each quarter. This gives four series of indices for each industry, and as the ratios from year to year for each quarterly series are the ratios of the latest to previous reported profits for the same companies, the change in the index depends solely on the true change in profits, not on the (slow) change in composition of the sample. Thirdly, these four independent series are combined into a single continuous series by what Prais [1957, p. 7] calls 'a process which is almost exactly analagous to seasonal "adjustment"': the mean annual trend for each quarterly series is calculated, together with the average of these four means, which is expressed as a quarterly figure. It is then assumed that the means of the four series should differ by this average proportional trend, and a correction factor is applied to each of the series (except that for the first quarter - taken as the base) so that this assumption is satisfied. The resulting continuous series is then used to interpolate annual figures of gross trading profits by industry (from NIE 35, discussed in Economic Trends July 1958 and March 1961), to which is added where appropriate (industries s and i) gross trading surplusses of public corporations (from NIE 39). (This is the one exception where final figures were not available by the end of 1967; figures for the year 1966 are from NIE for 1968.) Finally, this method is only applied to industries s and mfg; figures for m and n are obtained directly from these.

Figures for profits are open to many objections, for more than most other variables they are in practice dependent on the frequently incompatible accounting conventions of individual firms. Disaggregation by industry and quarter increases the possible hazards and thus less confidence may be attached to figures for profits than to most other series. In particular there are three main objections. Firstly, a basic conceptual difficulty, figures ascribed to a quarter are not 'true' figures of profits generated in that quarter but a four quarter moving average centered on the quarter; thus the figures are in effect seasonally adjusted and open to the dangers of this, as mentioned above. This is a serious defect in the series, which must limit the applicability of the study. There is however no way of improving the series for profits without new information, which is not available. Ideally official figures of quarterly profits disaggregated by industry, similar to the available aggregate figures, are required. Until such time as these are published the best that can be done is to construct more artificial series on the lines of the method used here. This rather circumspect attempt to derive series is considered more useful in this essentially short term disaggregated study than the evasion of the problem.

Secondly, the rate of growth of profits implied by the continuous series used to interpolate the annual data may be observed to be in higher nearly all cases than the rate of growth of the annual figures. This implies that <u>The Economist</u> sample is biased towards growth companies, which may be reasonable since its choice is restricted to public companies, which might be expected to grow more rapidly than the average, and within this restriction growth companies may be (subconsciously) chosen as being the most interesting. The implication of this is that the figures derived for the first and second quarters of each year might be expected to be too small, and for the third and fourth too large.

Thirdly, there is a basic objection to the use of a geometric linked series as suggested by Prais, that is where

$$(\text{index})^q_t = (\text{index})^q_{t-1}.(\text{latest profits})^q_t/(\text{previous profits})^q_t$$

when any profits figure is not strictly positive; for example it only requires one small loss (latest or previous) to turn all subsequent profits into losses, and a zero previous profit produces indeterminacy. This objection is of little practical importance where highly aggregated figures are used but can become very relevant where small groups of volatile companies are examined; although only giving aggregate figures Prais derives these from weighted averages of geometrically linked series for twenty industrial groups. Because of this the use of an arithmetically linked series, that is where

$$(\text{index})^q_t = (\text{index})^q_{t-1} + (\text{latest profits})^q_t - (\text{previous profits})^q_t$$

using a suitable base, or a weighted average of arithmetically and geometrically linked series, was considered. This would be justifiable conceptually if reported profits consist in part of planned remuneration to capital (interest) and in part of a residual (pure profit); the former might be expected to conform to a geometrically linked series indicating exponential growth while the latter might conform to an arithmetically linked one indicating the random nature of the residual element. This hypothesis may be tested for part of the data as the linked series of profits reported in the second quarter of each year should reflect the actual profits of the previous calendar year. The correlation between the true series and both arithmetically and geometrically linked series is thus examined for each industry, the results being given in table 2-3.

Table 2-3: Arithmetic and geometric series for profits

Industry	Squared correlation coefficient Arithmetic	Geometric
s	0·948	0·948
f	0·902	0·906
c	0·210	0·249
i	0·825	0·818
e	0·236	0·239
v	0·699	0·661
a	0·017	0·156
t	0·870	0·869
p	0·864	0·873
o	0·298	0·288

These results are far from conclusive, but slightly favour a geometrically linked series, which, with one minor modification is thus used. This modification concerns industry a for 1965 Q4, where the reported latest profit is negative; this observation is discarded, it being arbitrarily assumed that the proportional change for that period is the geometric mean of the proportional changes in the immediately preceeding and subsequent periods.

Finally, a check of the whole procedure is possible for industry s, where estimates of true figures of (aggregate) quarterly profits are obtainable from MDS 2 - with allowance for the change of status of the Post Office in 1961 Q2. These direct and derived figures are given (as they concern the central variable in the model) in table 2-4.

Table 2-4: Direct and derived total profits series, £m

Time	Direct	Derived	Time	Direct	Derived
1	821	732	23	988	1137
2	880	802	24	1010	1115
3	835	831	25	1062	1061
4	928	929	26	1160	1009
5	863	760	27	1095	1126
6	922	826	28	1126	1184
7	918	908	29	1003	1195
8	942	931	30	1274	1175
9	841	718	31	1191	1274
10	886	848	32	1355	1391
11	879	886	33	1337	1282
12	937	898	34	1441	1368
13	864	776	35	1345	1424
14	986	906	36	1426	1506
15	965	995	37	1492	1446
16	1090	1062	38	1510	1420
17	1059	927	39	1367	1469
18	1113	1104	40	1490	1527
19	1031	1141	41	1461	1419
20	1054	1140	42	1528	1363
21	1041	1014	43	1337	1346
22	1076	1056	44	1349	1480

Comparing these derived and direct figures indicates that the procedure adopted is at least acceptable - for the aggregate, which is all that can be checked. The trends are the same (naturally, due to the interpolation process) and more importantly the turning points in each series on the whole coincide; the main apparent difference is that the derived series fluctuates rather more violently, that is each peak and trough tends to be slightly exaggerated. There is no real evidence of the suggested underestimation of the first and second quarters and overestimation of the fourth. It should be noted that such a test can only provide an indication of the acceptability of the procedure; it is however all that is available.

Capital (K) Figures are of gross fixed capital stock, and are of necessity based on unpublished data (provided by courtesy of the Cambridge Department of Applied Economics); for this reason a slightly condensed matrix of the original figures is given in table 2-5 (below). These figures are however updated and revised versions of published material by the same authors, whose derivation is documented in CDAE [1964 April]. The figures provided are disaggregated by industry (twenty one categories of manufacturing and ten of nonmanufacturing) and by type of asset (buildings, plant and machinery, and vehicles), and are at constant (1954) prices; their derivation is briefly summarised here.

The basic method is that suggested by Redfern [1955]: an assumed life, T periods, is postulated for different types of asset (not necessarily the same in different industries) and figures for gross investment at constant prices, I, are obtained for each industry and asset over a period which is twice the life of each asset, though where this is not possible a base is estimated from figures derived by Barna [1957]. Now all capital of a given type purchased in period t ends its life, or is scrapped, in period t + T, so scrapping in period t is I_{t-T-1} and extensions to gross stock, that is gross investment less scrapping, is given by $I_t - I_{t-T-1}$. Extensions are merely a change of gross stock, K, so

$$K_t = K_{t-1} + I_t - I_{t-T-1};$$

alternatively gross stock is the sum of all extensions still existing, that is made over the past T + 1 periods, so

$$K_t = S_j(I_j - I_{j-T+1}), \qquad j = t-T-1,\ldots t.$$

If required and if the necessary depreciation assumptions are made net stock, K^*, may also be calculated; if for example proportional, or straight line, depreciation and zero scrap value are assumed depreciation in period t is K_t/T, net investment is $I_t - K_t/T$, and

$$K^*_t = K^*_{t-1} + I_t - I_{t-T-1} - K_t/T.$$

It should be noted that depreciation, an accounting concept, and scrapping, a physical occurrence, are fundamentally different: as CDAE [1964 April, p. 25] point out, 'depreciation represents a loss of asset-years, scrapping a loss of assets'.

The treatment of this basic data has three main stages. Firstly, as the figures provided end in 1965, and in some cases in 1964, the series are brought up to date using the method described above: figures for gross investment for each industry and asset at 1954 prices are obtained by deflating current price figures for investment from NIE by the price deflators implicit in NIE 54; investment T years before is frequently unavailable, and so it is assumed that scrapping remains at its preceeding year's level - the stability of scrapping in the CDAE figures over the last few years suggests that this is not unrealistic. Secondly, industries (where necessary) and asset types are aggregated; ideally it would be preferable to keep the three types of asset separate but this is not attempted here. Thirdly, all figures are expressed in terms of the average price level over the sample period rather than in terms of the 1954 price level to maintain consistency; this has little importance since it involves only multiplying by a scalar, derived from figures provided by the source and given in table 2-6. This is thus an exception to the general convention of using current prices, and is made partly because constant price values for capital stock are of particular relevance, but mainly because in this isolated case the primary figures are at constant prices so there is no question of introducing, just of inevitably retaining, possibly arbitrary deflators. Indeed the process of reflation would itself be open to error. Thirdly, the resulting figures are allocated to the fourth quarters of each year, and these are linearly interpolated to produce figures for the first, second, and third quarters. This procedure only gives data for <u>mfg</u>, from which figures for <u>m</u> are obtained; figures for <u>s</u> are based on those of total gross capital stock (excluding private dwellings, public dwellings, roads, and other public social services) from NIE 66, given for the years 1954, 1958, and 1961 to 1966 - intermediate observations are derived from linear interpolation. Similar NIE figures for <u>m</u> were also derived, and these, after conversion to 1954 prices, showed marked differences from the processed CDAE figures; accordingly the NIE figures for s are multiplied by the ratio of the CDAE estimates for m to the equivalent NIE figures. No obvious reason for the discrepancy is apparent; the NIE figures are made compatible with the CDAE figures rather than vice versa purely for convenience, since there is no immediately available basis for assessing their relative accuracies. This is however of minor importance since the ratio between the two sets of figures remains relatively stable. Figures for n are obtained by subtraction.

As current prices are the convention, and as the constant price series here have been multiplied by a scalar reflecting price changes, a method (based on the CDAE method) for converting the series from constant to current prices is mentioned. This, because of the aggregation of types of asset, necessarily assumes that the proportions between the three types of asset in each industry do not change over time; the price deflators used are weighted averages of price series for each of the three types of asset, the weights being the average proportion of each during the sample period. The indices for each type of asset are necessarily proxy series since they are derived solely from other series discussed here. The index for buildings, B, is an unweighted average of price for industry o (which includes the main building materials) and unit labour costs, earnings multiplied by labour divided by output, in industry n (which includes construction). The index for plant and machinery, P, is similar, being an average of price for industry m and unit labour costs in

Table 2-5: Basic series for capital, £ millions

	Industry								
Year	f	c	i	e	v	a	t	p	o
1955	1289·9	1172·3	1092·7	1674·1	989·1	608·1	2068·8	784·9	768·6
1956	1330·1	1283·0	1175·7	1749·9	1040·8	633·4	2077·5	820·3	810·6
1957	1377·3	1407·6	1266·1	1823·1	1090·7	660·8	2085·6	858·7	854·9
1958	1425·0	1528·5	1355·7	1893·4	1124·2	687·9	2079·8	887·9	897·4
1959	1467·7	1628·6	1434·8	1951·1	1153·2	721·9	2074·2	910·8	943·6
1960	1521·8	1713·8	1567·5	2032·4	1192·1	755·4	2085·0	942·3	1005·7
1961	1577·5	1816·5	1762·8	2121·9	1254·6	792·4	2104·1	976·3	1070·4
1962	1633·3	1913·9	1917·3	2200·7	1311·5	825·9	2112·4	1010·6	1137·2
1963	1695·8	1981·4	2000·7	2281·5	1364·9	852·7	2112·3	1042·6	1197·8
1964	1770·3	2076·6	2070·3	2374·3	1409·2	887·8	2143·7	1082·1	1272·7
1965	1856·5	2208·8	2136·5	2460·8	1479·1	922·3	2182·2	1129·8	1376·1
see note	91·3	40·7	22·1	67·2	21·3	21·1	40·6	23·7	74·6

Note: The last row gives figures for asset V for 1965 computed as described in the text; these are included in the figures in the main table, for the source only gives figures for assets B and P for that year. A figure of 470·7 is similarly included for asset B in industry o for 1965.

Table 2-6: Supplementary figures for capital

Industry	Average proportions B	P	V	Latest scrapping, £m B	P	V	Life, years
f	0·4088	0·5340	0·0571	13·3	18·6	15·6	27·5
c	0·2469	0·7339	0·0192	3·3	20·6	5·2	27·0
i	0·2250	0·7628	0·0122	4·5	20·1	3·0	27·0
e	0·3239	0·6483	0·0278	10·1	33·0	9·1	27·5
v	0·3314	0·6543	0·0143	7·3	11·3	1·4	24·4
a	0·3608	0·6145	0·0247	4·7	17·0	3·9	24·3
t	0·4163	0·5655	0·0181	22·5	38·9	7·2	28·0
p	0·3316	0·6488	0·0196	6·2	16·4	3·9	27·6
o	0·3646	0·5824	0·0530	6·2	9·2	9·9	27·3

Notes Lives for assets B and V are 50 years and 7 years respectively for all industries.

Price reflators, (average 1956-1965)/1954, for assets B, P, and V are 1·220, 1·253, and 1·056 respectively.

industry e. The index for vehicles, V, is simply the consumer price index for vehicles (see below). These three price series can only be justified as approximations, but are accepted in view of their relatively minor importance. The weights, together with other information on the figures supplies (latest scrapping, main price changes, and lives of different assets in different industries) are given in table 2-6. No figures are derived for depreciation or net stock; gross figures are the basic figures and if net figures are required they may be derived solely from these figures and other data given.

2-5 Economy series

This section discusses variables which are not directly attributed to particular industries; this does not however prevent disaggregation where it is desirable for other reasons.

Consumption (C) Figures for consumers' expenditure (and other series relating to this - consumer prices and tax rates) are divided into four categories: food, drink, and tobacco (corresponding approximately to the final output of industry f and identified accordingly), motor vehicles (v), clothing and footwear (t), and other (o). The essence of this disaggregation is the removal of three relatively homogeneous and easily defined groups from the whole, rather than the definition of four meaningful categories - hence the large residual. All figures are from MDS 5, those for category o being derived by subtraction.

Consumer prices (B) Indices of consumer prices for the relevant categories are from MDS 165, aggregated where necessary at the published weights; they are discussed fully in Ministry of Labour [1959]. From 1962 Q1 the published figures are (annually) linked indices, before this they are based on January 1956; the earlier figures are thus adjusted according to the ratio of the new series to the old in January 1962, no longer overlap being available. Category v is from the disaggregated figures of MDS 166, subgroup motoring and cycling.

Exports (N) Figures for exports (and other trade series - imports and world trade) are given in total, s, and the two main divisions of this, n and m. They represent the value of UK produce exported, measured free on board; figures are obtained from MDS 129, s directly, m by addition, and n by subtraction.

Imports (M) Imports, valued to include cost, insurance, and freight, are obtained in the same way as exports from MDS 129.

World trade (WT) Figures are quantum indices of world exports excluding Mainland China, Russia, and Eastern Europe; they are from the Monthly Bulletin of Statistics table 'Selected Series of World Statistics'. In 1961 Q2 the base changes from 1954 to 1958 and as no overlap of quarterly figures is available the series are spliced according to a regression of the old annual figures on the new over four years. Figures for n are weighted differences of figures for s and m, the weights being the values of these categories in 1958, obtained from the <u>Statistical Yearbook</u> summary table 12 (1966 issue).

<u>Bank rate (RB)</u> Figures for the Bank of England rediscount rate are from MDS 148, and are the average of the three months in the quarter; where the rate changes in the middle of a month the new rate (or average of new rates) is taken for that month.

<u>Hire purchase restrictions (HP)</u> These are minimum legal deposits for hire purchase contracts expressed as proportions of the purchase price. Figures are from the <u>National Institute Economic Review</u>, February 1967, appendix III, being the average of the three months in the quarter; where the rate changes in the middle of a month the new rate is taken for that month. The figures apply to the one most important grouping, vehicles, and are unweighted averages of the rates for its published components, motor cars and commercial vehicles.

Two observations may be made on this approach. Firstly, hire purchase contracts are regulated in two ways, minimum deposit rates and maximum repayment periods, and so the former alone will not give the total effect of institutional controls in this field. Simplicity however dictates that only one series be used if possible, and the high degree of collinearity between the two, due to the official habit of altering these two instruments simultaneously, suggests that this may be satisfactory; further, Ball and Drake [1963,

p. 191-192] have observed that 'variation of the deposit rate [is] on average roughly twice as important absolutely as the length of contract life in determining consumer spending on durable goods'. Thus deposit rates alone are used. Secondly, the use of an unweighted average of components is necessarily arbitrary, and can only be based on the concept of the source giving groups of similar importance; the use of total sales as weights would be misleading in that it would overlook the different use made of hire purchase finance for each group, and figures for hirings by group are not immediately available. The effect of the choice of weights is probably small: the rates for the two groups appear reasonably collinear. Connected with this is the narrowness of the scope; this is due to the need for simplicity and the relatively very high relevance of hire purchase controls to vehicles.

Income tax rate (TY) This is the average rate of tax on all income from employment. Like the profit and consumption tax rates (see below) it is an annual figure allocated to all quarters, which may possibly cause distortion from the true proportion as rates are (usually) altered near the end of the first quarter; distortion may also arise due to the fact that no account can readily be taken of the lag between the receipt of income (or consumption) and the payment of tax - this might be expected to be of greatest relevance to the series for profit tax rates.

The income tax rate series is obtained by dividing income tax and surtax (after 1962 allocated according to the breakdown in that year) charged on all income from employment (from NIE 51) by the total income from employment (NIE 1).

Profit tax rate (TZ) This is the average rate of tax on corporate income, obtained by dividing taxes on the income of companies and public corporations (from NIE 51) by total gross trading profits (or suplusses) of companies and public corporations (NIE 1). Again this is an annual rate allocated to all quarters.

Consumption tax rate (TC) Figures are annual proportions of consumers' expenditure absorbed by all forms of indirect taxation, and apply to the four consumption categories. They are derived by dividing annual figures for taxes on consumers' expenditure (from NIE 29) by the corresponding figures for consumption (NIE 27). No allowance is made for expenditure by foreign tourists in the UK or consumers' expenditure abroad.

Total indirect tax (A) Figures for total indirect taxes less subsidies are from MDS 1.

Government expenditure on goods (GG) This is total purchases on current account of goods and services (excluding direct labour but including other factor costs such as imputed rents) by government; it is obtained by subtracting government expenditure on labour (see below) from total public authorities current expenditure (from MDS 1).

Government expenditure on labour (GL) This is total direct expenditure on current account on wages and salaries by government, excluding wages and salaries paid in trading services; figures are obtained by interpolating an annual series (from NIE 48) according to the series of total public authorities current expenditure, from MDS 1.

Government transfer payments (GT) This is total current grants from public authorities from MDS 4; it consists of national insurance benefits, family allowances, assistance grants, and so forth (but excludes government interest payments).

Government capital expenditure (GK) This is total expenditure on capital account, that is gross fixed investment plus the change in the value of stocks held, by government; it is obtained by adding the series derived for investment for g to the first difference of that derived for stocks for g.

Other income (V) This relatively small residual series is gross income from rent and self employment, or all domestic income other than labour income and profits; figures are from MDS 2.

Gross domestic product (Y) Figures for the basic aggregate of gross domestic product at factor cost are from MDS 2.

Time Four time series are used; though not strictly data series, and not given in appendix B, since they require no observation, they are briefly mentioned here. They are the first quarter seasonal, QU, the second quarter seasonal, QD, the third quarter seasonal, QT, and time; the first three are discussed in section 2-1, the last is time in quarters measured from 1956 Q1.

figure is taken for both. The early part of the series is thus adjusted according to the coefficients of a regression of the new series on the old for an overlap period of ten quarters. The adjusted series to 1961 Q1 is then compared with available annual figures on the new basis (same source), and adjusted accordingly where necessary. It is unavoidable that figures for industry o do not correspond exactly to figures of exports for the residual industry.

Import prices (PM) Indices of unit values of imports for the three aggregates distinguished in import figures (s, n, and m) are obtained from MDS 164 in exactly the same manner as figures for export prices.

World prices (PW) Figures are indices of the prices of world exports, excluding Mainland China, Russia, and Eastern Europe, for the three categories of world trade (s, n, and m); they are in dollar terms, which, as the official parity remained unchanged over the time period, is equivalent to sterling terms. The series are obtained from the Monthly Bulletin of Statistics in an identical manner to those for world trade.

Input prices (PP) These are indices of the delivered prices of materials and fuel used in industries, exclusive of purchase taxes; they are from MDS 127, and are similar to the series for (output) prices but cover only industries m, f, c, e, t, and p. Until 1959 Q3 quarterly figures for industries c and p are not available so annual figures are interpolated according to the series for m; figures for industry _e_ are from a combination of mechanical engineering industries and electrical machinery industries at equal weights - no weights for inputs are readily available, which is why there is no residual industry. The year 1956 is treated in the same way as it is for the (output) price series.

Debenture yield (RD) This is the average redemption yield of debentures and loan stocks quoted in London from MDS 156; full details are given in Financial Times [1962]. Before 1965 Q1 these figures are not available and the flat yield of two and a half percent Consols (from the same source) is used, after being adjusted according to a regression of the new series on the old over the rest of the period - eight quarters.

Rate of duty on drink and tobacco (DD) This series, and the other duty series (rate of duty on oils), are relevant because of the large effect on a narrow front of prices of one form of indirect taxation - customs and excise duties. Both (annual) duty series are expressed as indices because of the difficulty of obtaining suitable deflators of tax revenues in equivalent units; deflators in physical units are used since the duties are levied at given rates per physical quantity, not ad valorem. In each case tax revenue figures for the fiscal year (to 5 April) from the Report of the Commissioners of Her Majesty's Customs and Excise, table 3 (1966 issue) are deflated by figures for the preceeding calendar year, implying a constant lag of approximately one quarter between consumption and the payment of tax.

The rate of duty on drink and tobacco series is obtained by dividing the net receipts of duty from spirits, beer, wine, British wine, and tobacco by the physical quantity consumed, that is consumers' expenditure at constant prices on alcoholic drink and tobacco, from NIE 28; it is assumed that all drink and tobacco is a final product.

Rate of duty on oils (DO) This series is similar to the rate of duty on drink and tobacco series, and is obtained by dividing the net receipts of duty from hydrocarbon (fuel) oils by the physical quantity of petroleum products delivered for inland consumption, from MDS 59.

2-7 Fixed proportions

The last three sections discuss all the relevant time series data. There are also three sets of fixed proportions relevant to the model, the first two being of rather nominal, and the third of more real, importance. These proportions do not form part of the data in the strict sense and thus do not appear in appendix B; rather they form part of the numerical specification of the model and are thus given in appendix D. They are discussed here for convenience.

Weights These reflect the relative sizes of the ten industries; they are basically the (1958) weights in the index of industrial production given by CSO [1959] and listed in table 2-1. This does not produce a weight for industry *n*, and this obtained from its contribution to gross domestic product in 1958 from NIE 11:

$$w_{\underline{n}} = w_{\underline{m}} \cdot \text{(product originating in } \underline{n}\text{)/(product originating in } \underline{m}\text{)},$$

where w_j is the weight for industry j. These figures, expressed as proportions so that their sum is unity, form the elements of the vector $\underline{w}$ in appendix D.

Bases These are the bases of the series of value of output, that is output multiplied by price. These series are both weighted for the year 1958 - though they are expressed in terms of (average 1956-1965) = 1, and accordingly the bases are calculated for that year. The base for each industry is obtained by dividing the value originating in terms of labour income (labour multiplied by earnings) plus profits by the index of value originating in terms of (average for the year) output multiplied by price. This process gives the vector $\underline{q}$ in appendix D.

Input-output coefficients The input-output matrix $\underline{A}$ given in appendix D is derived from CDAE [1963]; each element, a_{ij}, of this matrix gives the direct demand for the output of industry i by industry j per unit output of industry j for the year 1960 - at 1960 prices. The use of a fixed coefficient input-output matrix was discussed in chapter 1; the restrictions implied by this, discussed in for example CDAE [1963, chapter 1], are accepted not because it is maintained that the assumptions implied are wholly fulfilled but because the probable errors due to non-fulfilment are assumed to be an acceptable price to pay for the simplicity of the method. This then justifies the selection of a matrix typical of the time period; ideally some form of average matrix might be constructed, for example by making each element equal to the geometric mean of the corresponding elements in matrices for each of the years of the time period, such matrices being derived from any two base matrices by the 'RAS' method proposed by CDAE [1963, chapter 3]. This is not attempted mainly because (estimated) data for a year almost in the middle of the time period, 1960, is readily available, and also for simplicity.

Simple input-output matrices are based on a homogeneity assumption, that is they assume that each industry produces only one product, and industries are defined so as to ensure that no two industries make the same product, so that the make matrix is diagonal. More sophisticated analyses accept the nondiagonality of the make matrix, but make another less restricting assumption: either, in the terminology of CDAE [1963], that '<u>all commodities, whether principal or subsidiary, produced in one industry are made by the same process and therefore require the same input structure</u>' [p. 13] - the industry technology, or that 'technological processes depend on the nature of the individual commodities produced, and therefore that <u>inputs are determined not by the industry which absorbs them but by the commodity into which they enter</u>' [p. 14] - the commodity technology. The environment in which the matrix is to be used in this study dictates the use of the most simple (homogeneity) assumption, and as the CDAE matrix is based on the commodity technology it is adjusted accordingly.

The first part of this process makes the two disaggregation schemes compatible by condensing the CDAE make matrix [table 5] and absorption matrix [table 6] for 1960 at 1960 prices from order thirty one to order ten. The validity of the homogeneity hypothesis at this level of disaggregation may be investigated by examining the nonzero offdiagonal elements in the condensed make matrix, the results being summarised in table 2-6.

Table 2-6: Diagonality of the make matrix, £ millions

Industry/ commodity	Total output industry	Total output commodity	Industry output commodity
n	17408	17457	17339
f	2927	3029	2917
c	2822	2709	2633
i	2795	2799	2636
e	3858	3836	3621
v	2553	2515	2376
a	1858	1864	1706
t	3133	3134	3100
p	1443	1445	1436
o	1855	1864	1793
Total	40652	40652	39557

It may be seen that the offdiagonal elements are of relatively little importance in all industries, and the sum of all off-diagonal elements (which involves no cancelling) is only approximately 2·7 percent of the sum of all elements. Reasonable confidence may thus be attached to the homogeneity assumption.

If the validity of the homogeneity hypothesis is accepted an input-output matrix is readily derived from an absorption matrix by dividing each element in the latter by the total output of the relevant purchasing industry (or the total output of the commodity produced by that industry). Thus the condensed absorption matrix is treated in this way, a simple average of the total output of the relevant industry and commodity being used as the divisor. The resulting input-output matrix is given in appendix D.

CHAPTER 3 THE THEORETICAL MODEL

This chapter discusses the theoretical details of the model which was introduced conceptually in chapter 1 and forms the basis of this study; the discussion commences with the general structure then proceeds to the individual relations.

3-1 The model as a whole

As has been indicated in the two preceeding chapters the model is essentially one of industrial disaggregation; it is stochastic, dynamic, and has a time period of a quarter. More specifically, the model consists of ten industry submodels connected with each other and with a skeletal economy model by a number of horizontal and vertical links. The method consists of building an outline industry model which is applicable to all industries, modifying this to suit individual industries, and constructing an economy framework to connect the resulting components. The first of these three stages is perhaps the most important and is discussed in detail in this chapter; the third stage is also discussed, more pragmatically, here. The second stage is more empirical and must await the estimation of the model in chapter 4.

<u>Industry outline</u> The outline industry model consists of ten equations, eight stochastic and two deterministic, and thus involves ten endogenous variables; it also contains other variables which are endogenous to the complete model yet exogenous to the individual industry, and a number of purely exogenous and other predetermined variables. At the theoretical stage two alternative relations are postulated for the determination of profits, since profits are the essence of the model. Thus in effect two (overlapping) models are proposed at this stage, the inferior being discarded only after testing by prediction in chapter 5.

It is desirable to consider one variable or combination of variables as being dependent in each equation of the model, and thus as being determined explicitly. The equations of the outline model are thus chosen with the intention of forming a logically plausible whole before any attention is given to the detailed specification.

The first of these equations expresses the demand for the industry's output in value terms as the sum of the individual demands; parallel to this is a technological supply relation giving physical output as a function of factor inputs. Tying these two together is the important (in the short term) relationship determining the accumulation of stocks; it is in these first three relations that the short run productions decisions first manifest themselves. Turning now to the medium and long run respectively, the next two equations explain the change in the labour force, both employed and unemployed, and the change in capital stock, or gross investment. The first of these is basically a supply relation whereas the second is a demand equation - reflecting the respective status of the two factors in a capitalist economy. Price of course maintains equilibrium in the production equations, but is determined explicitly, by producers, in the next equation. The rewards of factors are now explained: labour earnings are determined by a rate of change equation reflecting mainly bargaining strengths, and profits are given either stochastically in terms of the overall prosperity of, and degree of competition in, the industry, or deterministically by the identity between income and product. Finally, two fixed proportions indicate the intermediate demand for the industry's product andthe material prices facing the industry.

3-2 Preliminary observations

This section briefly mentions a few general points on the construction of the model which are treated in a standard way, or elaborated in another chapter, and discusses two other points more fully; some conventions are also given.

One of the more important conceptual problems arising in connection with aggregate models, or indeed any macro-economics, is that of aggregation. The problem arises since most macro relations are based on inferences about micro behaviour and are in effect the aggregates of individual micro relations; further, individual relations may only be meaningfully aggregated if they are of a linear form, or reducible to this (by the use of logarithms for example) and many individual relations depend on relative levels of variables or other nonlinear combinations. The problem is by its nature insoluble - it may only be avoided by disaggregation, either permanently, or temporarily to determine the aggregation bias; the industrial disaggregation of this model may be expected to reduce, though not remove, aggregation error. The subject is fully discussed by Theil [1954] for example.

The links between individual industries and the whole are fundamental to the model; these and other relations are essentially simultaneous rather than recursive. The question of the choice of simultaneous or recursive structures is extensively covered by Wold - [1953] and elsewhere; its implications for this study are mainly statistical, and are discussed in chapter 4.

A quarterly model necessarily involves some form of explicit or implicit seasonal analysis; the treatment here is discussed in chapter 2.

One of the most obvious limitations of the scope of the model is its almost complete omission of monetary, as opposed to real, factors; indeed there is no attempt to explain any financial variables - the only two incorporated (bank rate and hire purchase restrictions) are exogenous policy variables and these have a minor role. This approach would not be justifiable in a model that sought to explain everything; it is perhaps valid here in that it defines more specifically the scope of the model - an analysis of real economic factors.

In the interests of consistency all industry models have the same number of equations, even though, as will be recalled from chapter 2, some industry variables are available only in partially aggregated form - stocks for [$\underline{e}$, $\underline{v}$, $\underline{a}$] and [p, o], and prices for [e, v, a, o]. In these cases the stocks for each industry of a group of n is taken as one n'th of the total, and the price for the industry is taken as being the price for the total; this rule is the simplest when there is no further information, and is thus adopted. The only reasonable alternative would involve using partially aggregated equations for these variables; if the relations were linear this would clearly be an equivalent treatment, though as they are not small differences may arise. A similar point concerns the treatment of the price of industry n, which is not readily available; this is defined as being the price for m, and so the standard price equation for this industry is replaced by this identity.

Values We now turn to the question of whether to use current or deflated values in the model. The emphasis on profits here means that the usual approach of working primarily in deflated values may well be inappropriate; partly because of the high dependence of profits on prices and partly because of the conceptual difficulties involved in deflating profits. This is illustrated by the study of profits discussed in chapter 1, where Evans [1968] finds that using money values rather than real values in equations for profits in twenty one industries explains on average an additional eighteen percent of the residual variance, and concludes [p. 356] that 'current dollar values are far superior to constant dollar values in explaining profits'.

There are three basic ways (which may be mixed) of treating this question. Firstly, and conceptually preferable, is the fullest treatment where all variables are expressed in both value and volume terms. This requires a large number of equations, including a price adjustment equation for each variable, often with associated inventory holding functions, and the identity that value is equal to volume multiplied by price. It is considered too complex practically in this disaggregated model.

Secondly, money values may be used. This is clearly inappropriate for purely technical relationships such as production functions, but is perhaps more appropriate than is sometimes supposed for many decision equations. It is observable that there is some money illusion, especially in the short run: if the pound is the standard unit of account then both producers and consumers will tend to base their decisions on pounds, rather than on '1958 pounds'. This is desirable, for the pound is a meaningfully defined unit whereas the '1958 pound' is not; the conceptual problems of defining meaningful price indices, particularly where there are quality changes and new goods, are well known. This applies to flows of measurable quantities of goods and services; for residuals and other nonflow variables the problems are greater, and of course at the practical level they are greater still for both categories. The use of money variables in demand functions may be criticised on the grounds that it is not consistent with the supposition that such functions are homogeneous of degree zero in prices. Apart from the question of whether this requirement should theoretically be fulfilled in the short run (and by accepting some money illusion we have suggested that it need not), this may be unimportant in practice for linear equations whose values cover a small range not far from zero, for as Christ [1956] has pointed out, the nonhomogeneous linear relation may be interpreted as an approximation to a nonlinear relation that is homogeneous.

Thirdly, there is the (usual) practice of using deflated values. This may be criticised on its treatment of the usual accounting identities, which strictly apply only to money relationships and not to deflated values. An example of the dangers of working with deflated variables appears in the Klein and Goldberger [1955] model referred to in chapter 1, as was observed by Christ [1956]. In this model consumption is deflated by a price index for consumer goods, while disposable income is deflated by an index for gross national product. As the latter increased relative to the former over the sample period, though only by about eight percent over twentythree years, the model gives deflated disposable income a downward bias relative to consumption. Thus serious errors in the predicted values of personal saving, obtained from the identity between the three variables, are produced by making the identity apply to deflated values. Saving then is too high at the beginning of the period and too low at the end, even becoming negative in one year. This sort of result from a mild change of relative prices strengthens the case for the use of money values.

This model uses a mixed stratagem, which is perhaps optimal for its objectives and the resources available. The detailed mix may be seen from the equations: the two physical relations (the production function and the intermediary demand equation) are expressed in deflated (or physical) values, and the other, behavioural, relations are expressed in money values.

Distributed lags This short term model, being essentially dynamic, makes much use of hypotheses concerning the expected values (not in the statistical sense) and desired values of variables, which are most conveniently explained in terms of distributed lags. To avoid repetition later this digression briefly examines two relevant interpretations of such transforms; though not of direct relevance at this stage some statistical implications are, for convenience, also mentioned here. The discussion originates from Koyck [1954]; a more recent coverage is given by Jorgenson [1966].

We assume that there are two variables, x, y, such that y is an exact linear function of the expected value of x, x^e, so

$$y = \alpha + \beta x^e \qquad (1)$$

where α, β are constants. We further assume that the expected value of a variable, x^e, is a weighted average of its actual value, x, and the expected value held in the preceeding period, x^e_{-1}, (that is of the last expectation and its materialisation) so

$$x^e = \gamma x + (1 - \gamma)x^e_{-1} \qquad (2)$$

where γ is a constant such that $0 \leq \gamma \leq 1$; if γ is zero the present expectation is merely the last expectation, if it is unity it is simply the actual value. Substituting (2) in (1) gives

$$y = \alpha + \beta\gamma x + \beta(1 - \gamma)x^e_{-1},$$

and substituting the lagged version of (1) in this gives

$$y = \alpha\gamma + \beta\gamma x + (1 - \nu)y_{-1}. \tag{3}$$

Alternatively we may assume that the desired level of y, y^d, is an exact linear function of x, so

$$y^d = \alpha + \beta x, \tag{4}$$

and that the adjustment of y to its desired level is imperfect, so that a fraction ν (again $0 \leq \nu \leq 1$) of the difference between the desired and actual levels is made up in a period; thus

$$y - y_{-1} = \nu(y^d - y_{-1}). \tag{5}$$

Substituting (4) into (5) gives

$$y = \alpha\gamma + \beta\nu x + (1 - \gamma)y_{-1}, \tag{6}$$

which is identical to (3). Thus these exact hypotheses using expected and desired values are mathematically equivalent.

We now assume that y is a linear function of the expected value of x and a random disturbance, u, so that

$$y = \alpha + \beta x^e + u; \tag{7}$$

we further assume the same type of expectations hypothesis though now involving a disturbance, v, so that

$$x^e = \gamma x + (1 - \gamma)x^e + v. \tag{8}$$

By a method of substitution analagous to that for the exact case, (7) and (8) give

$$y = \alpha\gamma + \beta\gamma x + (1 - \gamma)y_{-1} + (u + \beta v - (1 - \gamma)u_{-1}), \quad (9)$$

which is equivalent to (3) with a (composite) disturbance term. The disturbance term is however of interest for it will in general be serially correlated even if the two original disturbances are not, for it includes the term $u - (1 - \gamma)u_{-1}$, which is correlated with its previous value, $u_{-1} - (1 - \gamma)u_{-2}$, since both contain u_{-1} (unless $\gamma = 1$). Thus estimation of the transformed equation is more complicated than that of the original equation even if the latter is free from serial correlation, but if this already has positive serial correlation, which is relatively common in economics, estimation of the transformed equation may be simpler than that of the original equation. If the u's are serially correlated with a first order autoregression coefficient $1 - \gamma$, so that

$$u = (1 - \gamma)u_{-1} + w,$$

where w is a (serially independent) disturbance term, then $u - (1 - \gamma)u_{-1}$ is serially independent; this of course has zero probability, but the commonness of positive serial correlation (that is $1 - \gamma > 0$) in econometrics suggests that this transform may well reduce serial correlation.

It is important to note that the transformed equation includes y_{-1} as an explanatory variable, which raises further possible complications. If y is serially correlated so that y_{-1} is related to y then y_{-1} is not truly predetermined, and treating it as such may introduce simultaneous equation bias. This ia a matter of more general relevance, and is mentioned in chapter 4.

Alternatively we may assume an equivalent stochastic desired value, or imperfect adjustment hypothesis, so that

$$y^d = \alpha + \beta x + u \tag{10}$$

and
$$y - y_{-1} = \gamma(y^d - y_{-1}) + v. \tag{11}$$

By the same method as above (10) and (11) give

$$y = \alpha\gamma + \beta\gamma x + (1 - \gamma)y_{-1} + (\gamma u + v),$$

which is equivalent to (6) with a disturbance term, but is not equivalent to (9), for the disturbance term here does not introduce any serial correlation. It may be noted that neither model alters the nature of the distribution of the disturbance term, for if, as is assumed, these independently are normal any linear combination of them is also normal.

These are the two observationally equivalent transformations of this type used in the model; they may be shown to be two particular interpretations of the general distributed lag hypothesis that y is a function of x and all previous values (here in discrete time) of x such that the coefficients of lagged values are geometrically declining, so that

$$y = \alpha + \beta_0 x + \beta_1 x_{-1} + \ldots,$$

where
$$b_i = r\beta_{i-1}, \qquad i = 1, 2, \ldots$$

and r is a constant such that $0 \leqq r \leqq 1$. Thus

$$y = \alpha + \beta_0 S_i(r^i x_{-1}), \qquad i = 0, 1, \ldots$$

or
$$y_{-1} = \alpha + (\beta_0/r) S_i(r^{i+1} x_{-(i+1)}), \qquad i = 0, 1, \ldots$$

that is $y_{-1} = \alpha + (\beta_0/r)S_i(r^i x_{-i})$, $\quad i = 1, 2, \ldots;$

subtracting this from the first equation gives

$$y = \alpha(1 - r) + \beta_0 x + ry_{-1},$$

which is equivalent to (3) or (6) if r is replaced by $1 - \gamma$.

Conventions Equations are discussed individually, being identified and referred to by (for industry equations) the number and name (for identification only) given in their heading; economy equations are identified by the lower case letters (c, b, m, n, v, a) associated with their dependent variables (with primes where necessary to indicate various forms of a basic equation for different categories). The variables involved are represented by the symbols given in chapter 2 and repeated in the glossary in appendix B. For these, lower case letters refer to industry variables and upper case to economy variables; one letter variables are endogenous, two letter exogenous. In the interests of clarity a few three letter (always upper case) variables are used to define certain combinations of other variables; these definitions are all given in context. Parameters are represented by double subscripted α's, the first subscript referring to the equation (number for industry equations, letter for economy) and the second to the term, so α_{ij} is the parameter of the j'th term in the i'th equation. As in the preceeding subsection variables may have one subscript referring to the time period, the absence of which indicates the current period; thus x_{t-i} is written as x_i and x_t as x.

To promote clarity the constant, α_{i0}, seasonal terms, and residual in each equation i are combined in the term ω_i, so

$$\omega_i = \alpha_{i0} + \alpha_{i(j+1)}QU + \alpha_{i(j+2)}QD + \alpha_{i(j+3)}QT + \text{(residual in equation i)},$$

where there are j nonnormalised terms in the equation.

Unless it is indicated to the contrary all industry variables apply to any industry - they form the typical industry of the outline model.

It must be emphasised that as the subject matter of this chapter covers a large part of the content of macroeconomic theory the discussion is necessarily both brief and particular: it does not aim to present a synthesis of existing theory, but merely to indicate the more important theoretical bases of the individual equations, together with their limitations, some alternatives, and possible extensions.

3-3 Industry stochastic equations

1. Demand This, though important, is not in itself a particularly interesting equation; in a world of perfect data it would be an identity: total demand for an industry's product is the sum of all individual demands. The composition of many components of demand in terms of their industry origin are not however known, so this equation in effect estimates these proportions. Some attempts were made to treat this as a production decision equation of the type used in the model of Klein et al [1961] for example, which seeks to explain the production decision in terms of lagged final demand, acting as a proxy for stocks. These attempts were abandoned, despite

experiments using stocks explicitly, as there seemed nothing to be gained from postulating a rather tentative hypothesis concerning entrepreneurial behaviour when a logically safer approach would explain as much, albeit indirectly. This of course is only valid as a stock formation equation is inclined explicitly.

As the equation is a quasi-identity, variables are in money values. The most important components may be expected to be demands by other industries, followed by one or more types of final demand. This is an exception to the rule of generality in the outline model: one or more categories of final demand are selected according to a priori and empirical ideas on the demand structure facing each industry. Finally, total changes in stocks held by the industry are included as these constitute the demand by the industry for its own product to hold as stocks. This does not allow for the changes of stocks of raw materials, and conversely for stocks of the industry's product held by other industries, for 'intermediate demand' measures only 'intermediate consumption'. It is outside the scope of this model to investigate the changes in composition of stocks, though stocks of the industry's product held by other industries might be reflected by the total (that is all industries') changes in stocks. Other factors however might well swamp the one we wish to identify in such a formulation, and as preliminary experiments indicated that this happens this modification was not adopted.

The nature of this equation is perhaps unusual, and it should thus be emphasised that its main role is to act as an accounting relation rather than an explanation of economic behaviour. Its stochastic form is due to the lack of data, and emphasises the fact that estimates of data are just as stochastic as estimates of behavioural parameters. In practice however a line must be drawn somewhere and this more rigorous treatment is only applied to 'fixed' proportions such as these parameters which are not supplied as data in chapter 2.

Thus the form of the equation is

$$x.p = \alpha_{11}g.p + \alpha_{12}FDC + \alpha_{13}(s - s_{-1}) + \omega_1,$$

where FDC is one element (or two elements) of the set [c, I, GG, GK, N, M]; if FDC is two elements of the set then $\alpha_{12}FDC$ is replaced by $\alpha^1_{12}FDC^1 + \alpha^2_{12}FDC^2$.

<u>2. Supply</u> The (so called) supply function is a technological production function: output depends on inputs, all in physical units; it is central to the whole outline industry model.

This is a field in which the usual practice of linearisation may be inappropriate, for such a function implies that the marginal products of each factor are independent of the amounts employed, and that the elasticity of substitution (between two factors) is infinite, implying perfect substitutability. An alternative is the well known Cobb-Douglas function which is linear in logarithms; this implies declining marginal productivities of each factor and a finite eleasticity of substitution - though this is always unity. Perhaps the most important alternative is the homohypallagic function proposed by Arrow et al [1961]; this allows the elasticity of substitution to assume any (positive) constant value. The linear function has the practical advantage of simplicity, and also the conceptual one of being meaningfully aggregatable as it is additively separable; the other two functions do not share this property, though they are both separable in the multiplicative sense, and thus in the additive sense in logarithms. Preliminary experiments were made with the linear and (slightly modified) Cobb-Douglas functions, but as these were of equivalent acceptability the simpler (linear) function was adopted; this may be considered at least as a satisfactory approximation to some true function, particularly over a narrow range. As this equation is of basic importance the most relevant aspects of the adopted and alternative formulations, their implied marginal productivities, are given in the discussion of the estimates of the equation in chapter 4.

The production function then is linear in inputs, and the question of how to define inputs arises. The prime input is labour, which may be defined in many ways; ideally allowance should be made for hours worked (which can incorporate the distinction between part and full time labour), for quality, and for activity - that is whether engaged in production or overhead work. Of these perhaps the most important is hours worked, and the exclusion of this variable can only be justified by the need for compactness in the industry models. Hours and stocks are generally considered to be two of the most important peripheral (that is apart from the main output and income components) factors in short term models, and their usefulness in terms of explanatory power overlaps at least to some extent. After consideration of the general scope of the model it was decided that treating both to the desired depth was not feasible, and so hours are omitted. Stocks are preferred for inclusion partly because they are a component of gross domestic product, albeit a small one, and partly because of some difficulty in identifying work input, or even hours worked, with hours registered - as illustrated by Lithwick et al [1967]. Quality and activity are perhaps less important, and more difficult to measure - though the latter is successfully allowed for with US data by Kuh [1965]; both are omitted. The labour input then is simply the number of persons employed.

The second factor is capital, for which there are two important considerations: the use of gross or net stock, and allowance for utilisation. Net stock, by allowing for depreciation, attempts to take into account the lower contribution made to output by older stock. Clearly older stock is less productive than (established) new stock (or at the most equally productive - otherwise it would be reproduced), but there are great difficulties in calculating this effect. A statistical approach runs into identification problems if the result is to be used in a production function, and the use of arbitrary or accounting measures are clearly open to error - the latter particularly by exaggerating depreciation (even to the point of 'writing off' stock still in use and thus giving it an infinite average product). The ideal approach, of Domar [1961] for example, of treating each vintage of capital as a separate input is not practical here. Accordingly, and as the original data is in gross terms, gross figures are used.

This gives a figure for capital capacity, and it now becomes desirable to allow for capital not in use - which clearly cannot contribute to current output. There are four main ways of achieving this: adjustment according to the unemployment of labour (discussed by Solow [1957] for example), adjustment according to the consumption of some material input directly connected with activity - usually electricity (Jorgenson and Grilliches [1967]), the explicit use of full capacity output - for example the Wharton School method (Klein and Preston [1967]), and direct surveys - for example the McGraw Hill surveys (Eisner [1967]). The first of these is adopted here, mainly due to the fact that this needs no data not already required by the model; this gives a measure[1]

$$k_u = k_c . \text{ł}/(\text{ł} + u),$$

where k_u and k_c are capital in use and capital stock respectively. This assumes that the proportional utilisation of labour force and capital stock are equal. The use of electricity consumption is similar - it postulates a fixed ratio between capital input (or use) and another input; it is not used here because of the unavailability of data and the need either to explain electricity consumption or (unsatisfactorily) to leave it exogenous. The essence of the Wharton School method is the derivation of figures for full capacity output by plotting actual output, marking off

1. The variable represented by the lower case letter 'l' is written as 'ł' to avoid confusion with the numeral '1'.

cyclical peaks, and connecting these by straight lines; capacity utilisation is then given by the ratio of the actual output to full capacity output. The objection to this approach is not that it relies on subjective definition of peaks (or subjective rules for 'objective' definition) but that it is only of use in the sample period, for the height and timing of the next peak must be known to predict current output. Finally, survey data is clearly impracticable here. The capital input then is gross stock at the start of the period adjusted by the proportionate utilisation of the labour force. Using stock at the start of the period appears from preliminary experiments to be as acceptable as, and is simpler than, using average stock over the period.

A third factor of production is land; as usual this must be omitted due to lack of data. It is unlikely to be significant in a developed economy.

Allied to the main factors of production is their quality as embodied in technical change; a proper treatment of this subject is outside the scope of this study. Preliminary experiments were made with a time trend in the hope that this would at least reflect autonomous innovation, but were not pursued because of the high collinearity between time and capital stock, even capital in use.

Finally, there is the question of nonfactor, or material, inputs. Ideally net output, or value added, should be used as the dependent variable, when material inputs would clearly be irrelevant; however gross output is used, as a proxy for net output (as mentioned in chapter 2), and the omission of material inputs is justified to the same extent that the use of this proxy is justified. As Domar [1967, p. 472] has aptly pointed out, 'it takes some ingenuity to make potato chips without potatoes'; here however we are concerned with turning potatoes into potato chips, for which a cook and frying pan may replace ingenuity.

Thus the supply function is

$$x = \alpha_{21}l + \alpha_{22}k_{-1}l/(l + u) + \omega_2.$$

3. Stocks As was indicated in the discussion of the exclusion of hours, the accumulation of inventories is one of the more important variables in the explanation of short term fluctuations; it is also notoriously difficult to explain - as shown for example by Lovell [1964]. Stock formation here is treated as being basically voluntary (perhaps an undesirable assumption, but one which is usually necessary in macro work due to the difficulty of inferring the relative amounts that are voluntary and involuntary), and is explained by three factors: a modified accelerator effect, a speculative price effect, and the (interest) cost.

The well known simple accelerator approach depends on the hypothesis that a fixed amount of stocks must be held in order to produce smoothly a given output (here for simplicity in money values, which is equivalent to assuming that the price of stocks is proportional to the price of output), so

$$\ddot{s}/(x.p) = \alpha$$

where α is a constant such that $\alpha > 0$. Taking first differences over j periods produces the familiar form

$$s - s_{-j} = \alpha(x.p - x_{-j}p_{-j});$$

as stock adjustment is essentially a short term phenomenon a lag of one quarter is used, so j is unity. This simple form has various defects, as shown for example by Lovell [1964]. Perhaps the most important of these, particularly with quarterly data, is the omission of expectations, for it is clearly more logical to postulate that stock formation depends on the expected rather than the actual (and thus unobserved) change of output. Alternatively, making some concession to partially involuntary accumulation, intended rather than actual stock formation may depend on the change of output. Either of these important modifications may be expressed (as shown in section 3-2) in the form

$$s - s_{-1} = \alpha(x.p - x_{-1}p_{-1}) + \beta(s_{-1} - s_{-2})$$

instead of the form above, where β is another parameter such that $0 \leq \beta \leq 1$; this modification is adopted.

Another motive for stockbuilding is speculative: stocks may be added to because of an expected price rise, either for raw materials or output. Evidence on such behaviour is inconclusive and paradoxically, as shown by Lovell, shows less effect in disaggregated studies than in aggregate. The rate of change of price (of output) over the last quarter is however included as being a rather naive indicator of the best estimate of the change in price over the current quarter.

Inventory holding is frequently financed on short term credit, which is both relatively expensive and rapidly taken on or discarded. Thus the rate of interest on short term borrowing may be expected to affect inventory accumulation. Its practical effect is likely to be more pronounced for stocks accumulated for speculative purposes where more narrow and immediate margins are being considered; there may however be a lesser effect for stocks accumulated for production purposes, which is why the rate of interest is included as a separate variable, rather than combining it (by subtraction) with the price change variable. Bank rate is taken to represent short term borrowing rates as overdraft rates are effectively tied (linearly) to bank rate, and for a firm with (temporary) liquid funds the implicit interest cost may be taken as bank deposit rate, which is similarly tied to bank rate. No account is taken of general credit availability, which may be more relevant, since an analysis of monetary factors is outside the scope of this model.

The form of the equation for stocks is thus

$$s - s_{-1} = \alpha_{31}(x.p - x_{-1}p_{-1}) + \alpha_{32}p/p_{-1} + \alpha_{33}RB + \alpha_{34}(s_{-1} - s_{-2}) + \omega_3.$$

4. <u>Labour</u> This equation expresses the hypothesis that factor supply in an industry is determined by the relative remuneration of the factor in the industry; thus labour force is a function of relative earnings, and also of total labour availability. As this is a supply function the relevant dependent variable is total labour offered, that is labour plus unemployment.

Earnings rather than wages are used because of the rather arbitrary definitions of basic wage; this is particularly relevant to an industry study as the difference between basic wage and average, or even 'standard', earnings varies greatly from industry to industry. There are two relevant assumptions implicit in the use of earnings rather than wages: that there must be reasonable knowledge of earnings in different industries (or at least as good as that of wages), and that there should be no significant disutility of work in the relevant range - or an industry will not attract labour by offering high wages if this calls for a proportionately greater increase in work. This applies particularly as weekly rather than hourly earnings are used, for then the total amount of work as well as its possible unpleasantness is relevant. Though clearly not ideally satisfied these assumptions may well be acceptable in practice, or at least preferable to those implied by the use of wages.

The general availability of labour is of direct importance, and deflating the dependent variable by this instead of including it as an explanatory variable was considered but rejected because of the desirability of dividing the total labour force into those employed and those not employed - presumably the latter would be more mobile. It is relevant to mention here the rather arbitrary nature of figures for unemployment by industry, for these refer to those who were last employed in the industry, not necessarily those who are seeking employment in the industry. It may be possible to allow for this by a complex lag structure for unemployment based on figures of the average duration of unemployment, but this is not attempted.

Thus the labour equation is of the form

$$\bar{l} + u = \alpha_{41} e/E + \alpha_{42} L + \alpha_{43} U + \omega_4.$$

5. Capital Capital formation has long been recognised as being of prime importance in explaining medium term fluctuations and long term growth. There are perhaps two main hypotheses concerning the determinants of this, in the form of net investment demand: the accelerator approach and the profits hypothesis, both of which are discussed at length by Eisner [1964] for example. These, in modified forms, are combined in the capital equation.

The simple accelerator hypothesis is equivalent to that discussed above for inventory investment; it presupposes a fixed capital-output ratio, and is thus theoretically inappropriate when confronted with the typical excess capacity of cyclical decline, though, as Eisner shows, is a useful approximation and thus frequently acceptable in practice. The essence of investment is expectation since investment necessarily involves the future, and thus the accelerator is modified by a distributed lag expectations hypothesis, or alternatively this may be interpreted as an imperfect adjustment process to some desired level of capital stock (though this is more properly allowed for in the discussion of depreciation below). As with inventory investment the value of sales is used, but here more permanent changes are relevant, reflected by the change over the last year.

The profits hypothesis may either be interpreted as past profits being an indicator of future profits and thus of the marginal efficiency of capital, or as profits being a proxy for liquidity (either through being retained or through influencing possible lenders) on which investment depends. The choice of the specific form of the profits variable to be included is perhaps best left to preliminary experimentation; from this the last year's post tax profit is adopted. The deduction of tax is relevant whichever interpretation of the profits hypothesis is preferred; indeed if the scope of the model allowed it might be preferable to deduct dividends as well, leaving solely additions to reserves.

The above factors are relevant to the determination of net investment, and as the dependent variable is gross investment an allowance must be made for replacement investment, approximated by depreciation. This is assumed to be proportional to capital stock and so capital at the start of the quarter is included as an explanatory variable. An equally important reason for including the stock at the start of the quarter concerns a desired stock, or imperfect adjustment process, hypothesis: that there is a desired stock of capital which depends on the above factors, but only some fraction of the difference between this and the actual stock is made up in each quarter. If this fraction is represented by a constant α such that $0 < \alpha < 1$, and the desired and actual stocks by k^d and k respectively, then net investment i^n is given by

$$i^n = \alpha k^d - \alpha k_{-1}$$

and so capital at the start of the period has an inhibiting effect on net investment. This effect then works in opposition to the depreciation effect.

A possible criticism of this equation is the simple nature of the lags involved, which may not properly allow for a reasonable gestation period. Particular attention was paid to this point in preliminary experiments, but no conclusive evidence materialised for the use of longer or more complex lag structures. It may be relevant here that it is the large new investments which by their nature attract publicity, not the larger number of individually smaller routine investments which may be expected to mature more rapidly.

The cost of capital is not included as preliminary experiments showed that it was not significant, as may be expected with the small range of interest rates relative to expected returns. Thus the adopted form of the investment equation is

$$i = \alpha_{51}(x.p - x_{-4}p_{-4}) + \alpha_{52}LYP + \alpha_{53}k_{-1} + \alpha_{54}i_{-1} + \omega_5,$$

where $\quad LYP = S_j(z_{-j}(1 - TZ_{-j})), \qquad j = 1, 2, 3, 4.$

<u>6. Price</u> The price formation equation is a markup relation modified by the Marshallian scissors, the dependent variable being the wholesale output price.

The first element of cost is that of raw materials purchased. Preliminary experiments were made using input price (the observed variable) explicitly as an explanatory variable, and in turn explaining this as a function of the (domestic) materials price (derived from the input-output matrix and the vector of output prices) and world prices; this however achieved no overall increase in explanatory power and so the input price variable was omitted from the model, leaving only the materials price to represent the cost of nonfactor inputs.

The next element of cost is that of labour; this is taken as unit labour costs, that is the ratio of total labour payments to output. No attempt is made to investigate the hypothesis (a parallel of Friedman's well known consumption theory) that permanent rather than transitory changes determine markups, and that changes in basic wage rates are considered as permanent whereas changes in the difference between earnings and wages are transitory; this is one of the prices that must be paid for omitting basic wages from the model. The classical theory of price formation is essentially short run: past expenditure on capital is one of Jevons' byegones. Under imperfect competition capital costs may however be taken into account explicitly when fixing prices, especially in industries where these form a large part of total costs, so preliminary experiments were made taking these into account; capital costs were interpreted as depreciation costs, again represented by capital stock per unit of output. This had no significant effect.

The forces of demand and supply are taken into account though they do not appear explicitly; the excess of supply over demand is represented by the ratio of stocks to output at the start of the quarter. It may be valuable to take account of permanent and transitory factors here as well by using deviations from a trend; this was examined during preliminary experiments, but not found useful. An alternative measure of the relative strengths of supply and demand might be inferred from capacity utilisation, but the degree of confidence in the proxy for this adopted here prohibits its use other than for adjusting capital stock.

As the dependent variable is wholesale price no account need be taken of purchase taxes, since these are not levied directly on the producer. Excise duties are however levied on the producer and thus enter into the wholesale price; accordingly the rates of duty were included in preliminary experiments on the two industries for which they are relevant (<u>f</u> and <u>c</u>), but it was found, surprisingly (as they show reasonable variance over the period), that neither had a significant effect on price.

The adopted form of the price equation is then

$$p = \alpha_{61}h + \alpha_{62}\underline{l}.e/x + \alpha_{63}s_{-1}/x_{-1} + \omega_6.$$

<u>7. Earnings</u> The earnings equation is a condensation of what should ideally be a whole sector, explaining standard hours, basic wage rate, overtime, overtime rate, and bonuses, and connected by the relevant identities. If the omission of hours is accepted then the arbitrary nature of the relation between wages and earnings across industries justifies to some extent this condensation. Besides simplicity this has the advantage of representing a more continuous phenomenon, for typically wage bargaining is an annual process while the more fundamental variable, earnings, fluctuates more according to the factors affecting it. Salary rates are more open to individual bargaining and more responsive to economic factors than wage rates, and it is interesting in this context to note

the close correlation between salary earnings and wage earnings (but not wage rates) mentioned in chapter 2. As is usual rates of change are relevant, and so the rate of change of earnings is expressed as a function of four factors: unemployment, and the rates of change of output, of profits, and of consumer prices.

Unemployment is the basic explanatory variable for the wage element in earnings, as in the well known Phillips model. Both industry and total unemployment were considered; if there were perfect mobility between industries (this is not quite such a restricting assumption as mobility between occupations) the industry level would not be relevant, but this is not required. Preliminary experiments showed however that the industry figure (alone) is on the whole more important than the aggregate figure (alone), and because of the high degree of collinearity in most industries as satisfactory as both figures together; thus only the industry figure is used. This formulation conflicts with the findings of Schultze and Tryon [1965] with US data, possibly reflecting the different degrees of labour mobility in the two countries. It may be expected that the effect of a change in unemployment would be greater when unemployment is low than when it is high, partly because of underemployment when unemployment is high and partly because money wages very seldom fall whatever the state of unemployment; that is the rate of change of (money) wages has a floor at zero and so the rate of change of earnings may be expected to have a soft floor somewhere below zero. For this reason the level of unemployment is replaced by its reciprocal. It is relevant to note that unemployment does not perfectly reflect the more fundamental factor relevant here, the excess of supply over demand in the labour market. Ideally unfilled vacancies should be taken into account, but this is not attempted.

Just as unemployment may be relevant in explaining the wage element in total earnings, output may be relevant in explaining the hours element. This reflects the practice of using existing resources more fully during a period of temporary pressure on demand (or presumably during the early stages of a more permanent increase in demand), rather than increasing actual resources. The essence of this is that it is a short run phenomenon; in the medium run more labour is hired, in the long run more capital is used. Thus the rate of change of output over the last quarter is included.

Profits are included for two reasons. Firstly, they may be relevant in the bargaining process even if they are not known by labour, for the existence of high profits shifts the contract curve to labour's advantage by increasing the opportunity cost to capital of a strike; further, as negotiation becomes more centralised and frequently carried out under public and governmental surveillance, high profits tend to sway public opinion and government intervention to the side of labour. Secondly, it may frequently benefit the capitalist to maintain a generous remuneration structure where this can be afforded in order to attract high efficiency labour, or at least maintain a satisfied (and therefore quiet) labour force; this is particularly relevant where it is realised that abnormally high profit margins in oligopoly will attract new entrants, or possibly the attention of antitrust bodies. Profits then are included, in the form of the proportional increase over the last year, as quarterly changes are probably not known, or discounted, by the capitalist, and preliminary experiments suggested that a more complex lag structure would add little.

Finally, the inclusion of the change in consumer prices reflects the absence of (complete) money illusion in the labour market, where an increase in consumer prices may both raise the supply price of labour and swing public and official opinion to its side. This gives the earnings equation in the form

$$e/e_{-1} = \alpha_{71}/u + \alpha_{72}x/x_{-1} + \alpha_{73}z/z_{-4} + \alpha_{74}B/B_{-4} + \omega_7.$$

8b. Profits (b) This equation postulates a quasibehavioural explanation of profits based on the work of Evans [1968] discussed in chapter 1; it is one of the two alternatives proposed in the model.

The main determinant of profits is sales, for this provides the dividend to be shared between the factors of production, and where factor shares are relatively stable the total dividend of each industry may be expected to be the prime determinant of the share of each factor in the industry.

In an indirect examination of the determinants of profits Kuh [1965, p. 234] proposes that 'the basic determinants of cyclical variations in corporate profits arise from cyclical variations in labour productivity'. Labour productivity is included here indirectly through unit labour costs, since as Evans [1968, p. 348] points out, 'it is more reasonable to relate profits to changes in unit labour costs than to changes in productivity'. Evans' formulation however takes no explicit account of labour productivity or unit labour costs since a measure of capacity utilisation is included which acts as a proxy for the relevant part of labour productivity, or the part of labour costs that does not vary with output. This is mainly the cost of overhead workers who must be employed even when output is low (and also usually employed at a fixed level of remuneration), but also applies to production workers through labour hoarding due to the costs of hiring and dismissal. As no very reliable measure of capacity utilisation is contained in this model unit labour costs are included explicitly. It should be stressed that unit labour costs are not included as costs per se, but as a proxy for some part of productivity; thus as shown in chapter 1 an increase in unit labour costs need not be associated with a fall in profits - an increase of unit labour costs under conditions of full markup and low elasticity of demand will increase profits.

One reason why past sales may be expected to be relevant is that this term might reflect the (positive or negative) effect of lagged wage responses, as discussed in chapter 1. This becomes redundant because of the explicit inclusion of unit labour costs, though there are two more important reasons for its inclusion; firstly, that it may reflect increases in fixed costs resulting from capital expenditure stimulated by prior increases in sales, and secondly that it may allow for more firms being induced to enter the industry and the resulting increase in competition and loss of economies of scale. It is the second of these which is taken as being the more relevant here; the first could be included explicitly if required. The lag used then should reflect the time taken by outsiders to enter the industry, which may be expected to be of the order of a year; preliminary experiments confirm that this lag is appropriate.

It may be noted that our interpretation of profits makes any consideration of payments for interest or royalties irrelevant, for they are all part of the reward of fixed factors. Inventory valuation adjustments may however be relevant, for profits are (ostensibly) measured before providing for stock appreciation, that is including any capital gains or losses from holding inventories. As this formulation of the profits relationship is relevant to the profit derived from production rather than speculation an allowance for the change in value of existing inventories (though not of course the value of the physical increase) should be appropriate. Thus preliminary experiments were performed using $z - s_{-1}(p - p_{-1})$ instead of z as the dependent variable, but these resulted in a significant loss of explanatory power. This suggests that although reported profits should take changes in the value of existing inventories into account they probably do not (especially if values rise) due to the conservative and arbitrary nature of corporate accounting. It is thus assumed (here and throughout) that the change in values of inventories are not included in the profit figures used, and so all income terms are interpreted as being income from production alone (which is what is desired). This assumption is clearly not ideally satisfied, but it is considered preferable to the only viable alternative - that full account is taken of increases in the value of existing inventories.

This profits equation is thus of the form

$$z = \alpha_{81}x.p + \alpha_{82}\dot{\imath}.e/x + \alpha_{83}x_{-4}p_{-4} + \omega_8.$$

<u>Additional equation</u> Capital stock only appears in the model in lagged form, and thus being predetermined requires no explanation. It would however be desirable to explain capital stock in certain cases, such as prediction over more than one period; for this reason an equation is suggested, though it does not strictly form part of the model.

Gross stock may be expressed in terms of the identity

$$k = k_{-1} + i - SCR$$

where SCR is capital stock scrapped in the current period, as discussed in chapter 2. If scrapping is assumed to be some constant proportion α $(0 < \alpha < 1)$ of capital stock at the start of the period, plus a residual, this gives the equation

$$k = i + (1 - \alpha)k_{-1} + \text{(residual)}.$$

<u>3-4 Industry deterministic equations</u>

This section presents the three fixed proportions of the outline industry model at this stage; in this section the industry subscripts i, j are used since variables from many industries are relevant in the equations for each.

8a. Profits (a) This equation proposes a fixed proportion to explain profits, as an alternative to the quasibehavioural equation given in section 3-3. It should be noted that this need not prove more accurate in the context of the whole industry model for two reasons. Firstly, data inaccuracies (or the fact that the bases, q, are strictly estimates) mean that the equation itself need not hold identically in prediction. Secondly, small errors in the predicted values of the two quantities between which profits is the difference will entail a large error in the predicted value of profits, whereas the quasibehavioural formulation does not express profits as a residual difference between two similar series, and also is anchored to some extent by the predetermined lagged sales term.

Profits then are expressed as the difference between the value of total product and total labour payments; for industry i

$$z_i = q_i x_i p_i - l_i e_i$$

where q_i is the base referred to in chapter 2.

9. Intermediate demand The basis of this equation was introduced in the discussion of the linkages between industry models in chapter 1. More specifically, the intermediate demand for the output of industry i, g_i, is the sum of the demands for its product by all industries (including i); this demand by industry j is the output of industry j (adjusted by its weight to reflate it from an index number) multiplied by the input-output coefficient a_{ij} (the demand for the output

of i by j per unit output of j). Thus

$$g_i = S_j(x_j w_j a_{ij})$$

where w_j and a_{ij} are the weights and coefficients referred to in chapter 2, and the summation is over all industries.

10. Materials price The basis of this equation was similarly introduced in chapter 1. Ignoring imports, the average price of the materials purchased by industry i, h_i, is a weighted average of the (output) prices of all the industries (including i) which supply these materials, where the weights are the relative importances, that is a_{ji} for industry j where a_{ji} is an input-output coefficient as mentioned above. Thus

$$h_i = S_j(p_j a_{ji})/S_j(a_{ji})$$

where the summation is over all industries; the division of the figure for industry i by the constant $S_j(a_{ji})$ is included purely to make these prices of the same order as the output prices.

3-5 Economy stochastic equations

This section describes the various stochastic relations between aggregated economy variables or partially disaggregated variables not meaningfully attributable to particular industries; these are divided into three categories - the consumer sector, foreign trade, and purely empirical relations. It should be emphasised that the nature and aims of the model imply that these relations are treated in less depth than those of the industry models.

<u>Consumer sector</u> Consumer expenditures are divided into the four categories discussed in chapter 2: food, drink, and tobacco; motor vehicles; clothing and footwear; and other. For all these categories the consumption function is of the well known Keynesian form with allowances for expectations, relative prices, and (where relevant) financial restrictions.

The prime determinant of most categories of consumption is disposable income. Preliminary experiments concerning the appropriateness of different types of income were made, particularly as regards its distribution between factors. Two approaches were investigated: treating property and labour income as separate variables, and taking total income as the income variable with an additional term expressing the interfactor distribution of income (taken as the ratio of labour to property income). The results of these experiments were inconclusive, and did not support the adoption of anything more complex than disposable labour income as the income variable; this is total (private plus government) earnings plus transfer payments, all after tax. For the motor vehicle category even this did not appear to be important, and was omitted.

Expected income may well be a more important determinant of consumption than actual income, and so a simple distributed lag expectations formulation is adopted (except for the clothing and footwear category where preliminary experiments indicated that it was not relevant). No attempt is made to separate Friedman's well known transitory and permanent components of income, or to use other time oriented effects such as the life-cycle or ratchet hypotheses; experiments were however made using the last four (rather than one in the distributed lag hypothesis) quarters' consumption on the grounds that this might indicate some normal level of consumption from which adjustments to current conditions are made, but this modification appeared unimportant. Such a modification was considered for the consumption function but not for the fixed and inventory investment equations in the industry models because of the relatively stable time path of consumption.

Relative prices may be expected to be important when examining categories of consumption, and thus the ratio of the consumer price of the category in question to the general consumer price level is included (except for the food, drink, and tobacco category, where preliminary experiments indicated that it was not relevant).

Finally, an exogenous policy variable, hire purchase restrictions, is included to take account of credit restrictions where they may be particularly important - for the motor vehicle category. Thus the consumption functions are

$$c = \alpha_{c1}DLI + \alpha_{c3}c_{-1} + \omega_c$$

for food, drink, and tobacco,

$$c = \alpha_{c'2}b/B + \alpha_{c'3}c_{-1} + \alpha_{c'4}HP + \omega_{c'}$$

for motor vehicles,

$$c = \alpha_{c''1}DLI + \alpha_{c''2}b/B + \omega_{c''}$$

for clothing and footwear, and

$$c = \alpha_{c*1}DLI + \alpha_{c*2}b/B + \alpha_{c*3}c_{-1} + \omega_{c*}$$

for the residual category, where

$$DLI = (L.E + GL + GT)(1 - TY).$$

Consumer prices are divided into the same four categories and are determined by a simple markup process, allowing for the effect at this stage of purchase taxes (though as preliminary experiments indicated that this was not relevant in the residual category it was omitted there). Ideally the wholesale-retail markup should be treated in more depth as the value added by the retailing industry, but the aggregation within nonmanufacturing makes this impractical here. Thus the consumer price equations are of the form

$$b = \alpha_{b1}p + \alpha_{b2}tc + \omega_b$$

for all categories except the residual one, and

$$b = \alpha_{b'1}p + \omega_{b'}$$

for that category, where the prices p are those of the corresponding industries (that is f, v, t, and o respectively).

Foreign trade Imports are divided into two categories, manufactures and nonmanufactures, and the demand for each is expressed as a simple function of expected domestic income, using a distributed lag formulation. Preliminary experiments were made treating imports of nonmanufactures as complementary to domestic production and thus dependent on various types of industrial output, but this produced no improvement. Experiments were also made incorporating the prices of domestic goods relative to world prices, but perhaps surprisingly this was not significant at this level of aggregation. The import demand functions for both categories are thus of the form

$$M = \alpha_{m1}Y + \alpha_{m2}M_{-1} + \omega_m.$$

The export demand functions are similar to those for imports: exports are divided into the same two categories and are expressed as a function of expected foreign purchasing power, represented by world exports and a distributed lag term (though this is omitted for manufactures where preliminary experiments indicate that it is not relevant). An exception is made to the use of solely money values in behavioural relations here, for world trade is in quantum units; despite the theoretical inconsistency this is considered preferable to the use of money values where the price is an external price affected by inflation and currency revaluation throughout the world. Export prices relative to world prices were investigated, but, as with relative prices in the import demand functions, were not found to be significant. The export demand functions then are

$$N = \alpha_{n1}WT + \alpha_{n2}N_{-1} + \omega_n$$

for nonmanufactures, and

$$N = \alpha_{n'1}WT + \omega_{n'}$$

for manufactures.

It should be mentioned that the partially autoregressive (in effect) nature of the equations seeking to explain the foreign trade sector would make them inappropriate for many uses; they are considered sufficient only in the wider context of the whole model.

Empirical relations There remain two minor yet necessary stochastic relations which can only be treated here on an ad hoc basis: one explaining other income and the other determining the market price to factor cost adjustment.

Income from rent and self employment (other income) is expressed as a function of the two main forms of income, total earnings and profits; thus

$$V = \alpha_{v1}L.E + \alpha_{v2}Z + \omega_v.$$

Total net indirect taxes are required to convert gross domestic product at market prices to gross domestic income at factor cost; they are expressed as a function of the total indirect tax on consumption, so

$$A = \alpha_{a1}TCT + \omega_a$$

where $$TCT = S_i(c_i tc_i),$$

the summation being over the four categories of consumption.

3-6 Economy deterministic equations

The economy deterministic relations fall into two groups of identities: those expressing aggregate figures as the (weighted) sums or averages of the corresponding industry components, and the aggregate accounting identities.

The first of these are self explanatory; they are

$$X = S_i(w_i x_i), \qquad P = S_i(w_i x_i p_i)/X,$$

$$L = S_i(l_i), \qquad E = S_i(l_i e_i)/L,$$

$$C = S_i(c_i), \qquad B = S_i(c_i b_i)/C,$$

$$I = S_i(i_i), \qquad S = S_i(s_i),$$

$$U = S_i(u_i), \qquad Z = S_i(z_i),$$

$$N = N_{\underline{n}} + N_{\underline{m}}, \qquad M = M_{\underline{n}} + M_{\underline{m}},$$

where the summation is over all industries (or categories for C and B), and the suffixes for M and N refer to their two categories.

The second group consists of an equation expressing total expenditure as the sum of each of the components of demand, reduced to a factor cost basis,

$$Y^e = C + I + GG + GL + GK + N - M + S - S_{-1} - A,$$

and an equation expressing total income as the sum of each component of income,

$$Y^i = L.E. + GL + Z + V.$$

Gross domestic product may also be expressed in terms of the aggregate of production, but this is not used here.

Strictly speaking the definitions of new (three letter) variables in the preceeding sections are also identities, but these are not included in this section as their sole function is clarity.

The model is now exactly determined, having the same number (128 excluding the definitions referred to and one of the alternatives for the profits equation in the industry models) of equations as endogenous variables. If a consistency criterion is required the model may be overdetermined by the addition of a further equation, most appropriately that representing the desired equality between total income and total expenditure (which are not defined as being identical), that is

$$Y^i = Y^e \ (= Y).$$

This is taken up in chapter 5.

3-7 Summary of the outline industry model

The equations of the outline industry model are for convenience summarised here; they apply to all industries, though the industry subscripts i and j are only used where necessary - in the deterministic equations. Equations (8a) and (8b) are alternatives.

Demand $$x.p = \alpha_{11}g.p + \alpha_{12}FDC + \alpha_{13}(s - s_{-1}) + \omega_1 \qquad (1)$$

supply $$x = \alpha_{21}\ell + \alpha_{22}k_{-1}\ell/(\ell + u) + \omega_2 \qquad (2)$$

stocks $$s - s_{-1} = \alpha_{31}(x.p - x_{-1}p_{-1}) + \alpha_{32}p/p_{-1} + \alpha_{33}RB + \alpha_{34}(s_{-1} - s_{-2}) + \omega_3 \qquad (3)$$

labour $$\ell + u = \alpha_{41}e/E + \alpha_{42}L + \alpha_{43}U + \omega_4 \qquad (4)$$

capital $$i = \alpha_{51}(x.p - x_{-4}p_{-4}) + \alpha_{52}LYP + \alpha_{53}k_{-1} + \alpha_{54}i_{-1} + \omega_5 \qquad (5)$$

price $p = \alpha_{61}h + \alpha_{62}l.e/x + \alpha_{63}x_{-1}/s_{-1} + \omega_6$ (6)

earnings $e/e_{-1} = \alpha_{71}/u + \alpha_{72}x/x_{-1} + \alpha_{73}z/z_{-4} + \alpha_{74}B/B_{-4}$
$+ \omega_7$ (7)

profits (b) $z = \alpha_{81}x.p + \alpha_{82}l.e/x + \alpha_{83}x_{-4}p_{-4} + \omega_8$ (8b)

profits (a) $z_i = q_i x_i p_i - l_i e_i$ (8a)

intermed. demand $g_i = S_j(x_j w_j a_{ij})$ (9)

material price $h_i = S_j(p_j a_{ji})/S_j(a_{ji});$ (10)

where LYP = $S_k(z_{-k}(1 - TZ_{-k}))$, $k = 1, 2, 3, 4$,

and FDC is an element of [c, I, GG, GK, N, M].

CHAPTER 4 ESTIMATION AND THE ESTIMATED MODEL

This chapter concerns the estimation of the model specified in chapter 3 from the data presented in chapter 2, and has two main parts: the first concerns the nature of the method of estimation, its properties, and the theory of appraisal of the estimates that it produces, while the second discusses the numerical estimates obtained for the parameters of the model, particularly the adequacy of the outline industry model and its modifications. The numerical estimates themselves are given in appendix D.

4-1 Theory of estimation

Estimation is the essence of econometrics, for the stochastic nature of econometric models implies that different methods of estimation will produce different results; the model and the method of estimation thus define the (estimated) structure. The method of estimation of this model is then fundamental to the study, and is discussed in some conceptual detail in this section.

Maximum likelihood The nature of estimation of a model from some sample period data is the determination of the most likely values of the true parameters of the structure, where true parameters are population parameters - which can never be known (in a time series study) for if the model is to be general the population is not usually finite. An alternative interpretation of estimation is the determination of the values of the parameters which are most likely to generate the observed sample of data; this interpretation, the maximum likelihood method, is adopted.

More formally, let $f(x; \alpha)$ be the frequency function of the random variable x when α is the value of the parameter to be estimated; if $x_1, \ldots x_n$ are the values of x corresponding to each of the n (independent) observations on this variable, then the joint probability of observing these values (in order) when the parameter is α, $L(x_1, \ldots x_n; \alpha)$, is the product of the individual probabilities, so

$$L(x_1, \ldots x_n; \alpha) = f(x_1; \alpha) \ldots f(x_n; \alpha)$$

and the function L( ) is the likelihood function. The maximum likelihood method then takes the x's as fixed and finds the value of α which maximises this function, now written $L(\alpha)$. A similar procedure is followed where there is more than one parameter.

This method may readily be shown, as it is by Johnston [1963] for example, to be equivalent to the well known ordinary least squares method, which in effect chooses the value of α that minimises the sum of squares of the errors of the predictions of the x's.

The simple maximum likelihood method then concerns the conditional distribution of just one variable, given the values of a set of other (predetermined) variables. It is not however appropriate where there is more than one jointly dependent variable, for in such cases it is the joint conditional distribution given the values of other (predetermined) variables that is relevant. This is the type of model that this study is concerned with, and it may be expressed in the form[1]

$$\underline{By}_t + \underline{Gx}_t = \underline{u}_t$$

where $\underline{B} = [\beta_{ij}]$($\underline{G} = [\gamma_{ij}]$) is the g x g (g x h) matrix of coefficients of the g (h) endogenous (predetermined) variables, $\underline{y}_t$ ($\underline{x}_t$) is the g x 1 (h x 1) vector of the t'th observation on the g (h) endogenous (predetermined) variables, and $\underline{u}_t$ is the g x 1 vector of residuals corresponding to the t'th observation in the g equations. The reduced form of the model expresses $\underline{y}_t$ as an explicit function of $\underline{x}_t$ and $\underline{u}_t$, and may be written

$$\underline{y}_t = \underline{Px}_t + \underline{v}_t$$

where $\underline{P}$ ($= -\underline{B}^{-1}\underline{G}$) is a g x h matrix of reduced form coefficients and $\underline{v}_t$ ($= \underline{B}^{-1}\underline{u}_t$) is a g x 1 vector of reduced form disturbances.

The relevant joint conditional distribution may be obtained by rewriting the reduced form equations as explicit functions for all the v_{it}, obtaining the joint normal distribution of the v_{it}, and substituting for each v_{it} in this its value from the reduced form, giving a joint conditional distribution of the y's given the x's and reduced form parameters. The reduced form parameters are then expressed

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1. For typographical reasons upper case 'γ' is written as 'G'.

in terms of the structural equation parameters, and this becomes the likelihood function of the (structural equation) parameters given the observed values of the y's and x's. This rather lengthy process may be shown (for example by Christ [1966] - though in a different notation) to give the likehihood function

$$L = \frac{\exp(-\frac{1}{2}S_t(\underline{y}'_t + \underline{x}'_t\underline{G}\underline{B}^{-1})\underline{B}\underline{S}'^{-1}\underline{B}'(\underline{y}_t + \underline{B}'^{-1}\underline{G}'\underline{x}_t))}{(2p)^n(\det \underline{B}'^{-1}\underline{S}'\underline{B}^{-1})^{n/2}}$$

where $t = 1, \ldots n$ and $\underline{S}$ is the variance-covariance matrix of the structural disturbances[2]:

$$\underline{S} = E(\underline{u}_t\underline{u}'_t).$$

The maximum likelihood process then finds the $\underline{B}$ and $\underline{G}$ that maximise the value of L (or as this is simpler, of log L since this is a monotonic increasing function of L) subject to all the a priori restrictions. This however is a cumbersome process in practice (unless all the equations are just identified - when it is unnecessary) and so a compromise is frequently made whereby only a part of the a priori restrictions is taken into account. At the limit there is the case where no a priori restrictions are taken into account; this is equivalent to estimating each equation in isolation, and is thus the method of ordinary least squares.

Thus there are two extreme methods for estimating the parameters of a system of equations: (full information) maximum likelihood (FIML) and (no information) ordinary least

2. E( ) is the expected value operator; the (summation) operator S_t( ) should not be confused with the matrix $\underline{S}$.

squares (OLS); there are also a number of so called limited information maximum likelihood methods betwemn these two extremes. Before examining these however it is appropriate to discuss the concept of identification.

Identification Identification may be defined as the process of determining the parameters of the structure from those of the likelihood function. It is logically prior to the estimation process itself, and may pose complex problems; it is only briefly discussed here since, as will become apparent, this model is substantially overidentified. The identification problem arises because we are concerned with relations between jointly endogenous variables; the fact that these variables are jointly endogenous implies that they are related in more ways than one. Thus the problem is how to be sure that we are estimating the parameters of the structural equation we intend rather than some other relation between the endogenous variables, that is how to identify which relation we are estimating. Any model may be written in its reduced form, and the parameters of this may be estimated without identification difficulty since there is only one relation - if there were not the 'predetermined' variables would not be truly predetermined. The reduced form may not however always be transformed back into the structural equations, and thus, as we can encounter an identification problem with the latter but not the former, we may interpret the identification problem as that of deducing the values of the structural parameters from the reduced form parameters. Johnston [1963] shows that this interpretation is equivalent to the definition above, for the likelihood function may be written in terms of any observationally equivalent structure, for example the reduced form, thus

$$f(\underline{y}_1, \; . \; . \; . \; \underline{y}_n; \; \underline{x}) = f(\underline{v}_1) \; . \; . \; . \; f(\underline{v}_n),$$

and as $\underline{v}_t = \underline{y}_t - \underline{Px}_t$,

the parameters of the likelihood function are now the elements of the matrix of the reduced form coefficients together with the elements of the reduced form variance-covariance matrix $\underline{B}^{-1}\underline{S}\;\underline{B}'^{-1}$.

Thus the identification problem becomes one of making sure that $\underline{P}$ (or $\underline{B}^{-1}\underline{S}\;\underline{B}'^{-1}$) is such that the reduced form produces only one structural equation (or strictly one linear combination) complying with the restrictions placed on the relevant equation; thus identification may be achieved by imposing restrictions on $\underline{P}$ (or $\underline{B}^{-1}\underline{S}\;\underline{B}'^{-1}$). In practice the most important restrictions are those on $\underline{P}$, and of these the simplest are those of a zero or nonzero nature, that is specifying that certain elements in $\underline{B}$ and $\underline{G}$ are zero, or that certain variables appear only in certain equations. Koopmans [1949] has investigated this type of restriction and shows that a necessary condition for identification of an equation is that the number of variables excluded from the relation is at least equal to the number of equations less one. Expressing the number of endogenous and predetermined variables appearing in the relevant equation as g^* and h^* respectively, this condition may be written as

$$(j =)\ g - g^* + h - h^* \geqq g - 1,$$

or $$h - h^* \geqq g^* - 1,$$

that is the number of predetermined variables excluded from the relation must not be less than the number of endogenous variables included less one. If $j < g - 1$ the equation is underidentified, if $j = g - 1$ the equation is just identified, and if $j > g - 1$ then it is overidentified. This order condition is not a sufficient condition for identification; Koopmans also produces a necessary and sufficient rank condition, but this uses determinants involving the true values of the structural parameters, which are of course unknown. In practice the necessary condition is usually deemed to be sufficient, for it is extremely unlikely that the order condition would be satisfied without the rank condition also being satisfied; if the structural parameters are considered continuous then there is zero probability that the determinants which would need to be zero for this to happen would be zero - though they may be near zero, in which case the structural coefficients are illdetermined.

This discussion concerns the identification of a linear model, and as is shown by Christ [1966] for example, this may not be directly applicable to a nonlinear model. Christ also notes however that if the model contains at least one exogenous variable (that is excluding the constant term) then the conditions mentioned above apply, and also that by in effect forming new exogenous variables, nonlinearities may aid identification.

It is then apparent from the order condition that all the equations in this model are substantially overidentified, mainly due to the use of lagged variables and nonlinearities. Koopmans and Hood [1953] have pointed out the desirability of testing the hypothesised conditions for identifiability since the a priori exclusion of variables is itself subject to uncertainty, and suggest methods for doing this. This is not usually attempted in practice; it is omitted here because of the particularly strong prima facie case for overidentification.

Limited information methods Having briefly discussed identification we now turn to the range of estimation methods in the range between FIML and OLS known collectively as limited information methods. There is a wide range of such methods and thus discussion is limited to two of the most important - limited information maximum likelihood (LIML) and two stage least squares (TSLS).

The LIML method is so called because it finds values of the parameters that maximise the value of the likelihood function subject only to the a priori restrictions imposed on the equation being estimated. Koopmans and Hood [1953] have shown that this constrained maximisation is equivalent to minimising the ratio, m, of the unexplained variance of $\underline{y}^*$ in its regression on all the predetermined variables included in the equation being estimated, to the unexplained variance of $\underline{y}^*$ in its regression on all the predetermined variables in the model, where $\underline{y}^*$ is a synthetic variable combining the g^* endogenous variables appearing in this (the i'th) equation,

$$y_t^* = \beta_{i1}y_{1t} + \ldots \beta_{ig*}y_{g*t}, \qquad t = 1, \ldots n.$$

The ratio m cannot be less than unity, for adding explanatory variables cannot worsen the fit, that is increase the residual variance; at the extreme the added variables may have no effect, and m = 1. This is an explanation of the rationale of the method, for in seeking to minimise m it seeks to bring it as near to unity as possible, that is to find a vector $\underline{\beta}$ such that the predetermined variables omitted from the equation should make a minimal improvement in the fit of $\underline{y}^*$. The method is thus equivalent to the least variance ratio principle, which seeks to minimise m directly. It is central to the 'limited information' methods, for it may be shown that TSLS is a special case of FIML where m is set to unity, and also that OLS is a special case with m set to zero (this is not consistent with the definition of m, but then OLS is not a consistent method); these are some of the members of the family of k-class estimators developed by Theil [1961], Theil's k being equivalent to our m.

Though the TSLS method may be thought of as a special case of LIML its rationale is that the inconsistency of OLS estimation arises through the correlation between the disturbance term and the endogenous variables, so consistent estimators may be obtained by removing the stochastic component associated with the disturbance from the endogenous variables. To achieve this one of the endogenous variables is selected as the dependent variable for each equation separately, and all the remaining endogenous variables in the equation are replaced by their estimated values in terms of all the predetermined variables, calculated from the least squares estimates of the reduced forms; the method of OLS is then applied to the modified structural equation. Clearly there is a basic similarity between TSLS and LIML for both use all the predetermined variables in the model but do not require knowledge of the specifications of any equations other than the one being estimated; that is they utilise only the a priori information on that equation.

Assessment The evaluation of the various estimation methods available is in two stages. At first it is assumed that the usual estimation requirements are satisfied: more specifically that there is no serious multicollinearity, serial correlation of disturbances, correlation between disturbances in different equations, or heteroscedasticity, and that there are no specification errors. At this level we start by examining asymptotic properties, not because an infinite sample is envisaged, but because they are relatively tractable mathematically and may be useful approximations to small (or at least finite) sample properties. The asymptotic properties we shall take account of are consistency, normality, and efficiency. An estimator b of a parameter whose true value is β is asymptotically consistent if for all β the probability limit of b is β; this is not equivalent to asymptotic unbiasedness, which requires that $E(b) = \beta$ in the limit, for the mean of the limiting distribution may not exist, in which case unbiasedness is not defined. The estimator is asymptotically normal if the limiting distribution $f_n(b)$ as n (the sample size) tends to infinity is normal. The estimator is asymptotically efficient if it is consistent and normal and the variance of its limiting distribution is no greater than that of any other consistent and normal estimator. It may be shown, for example by Christ [1966], that OLS is inconsistent (thus its efficiency is not defined though by definition it has least variance) and TSLS, LIML, and FIML are all consistent, normal, and efficient (defined for the first two relative to other estimators using the same incomplete a priori information). Thus apart from a possible reservation about OLS, asymptotic properties are of little help in the choice of a method for estimating this model.

Even if an examination of asymptotic properties were to give a clear ranking of estimation methods this would not necessarily reflect the ranking of finite sample properties; further, asymptotic properties are frequently not defined for they depend on the limiting distribution having finite first and second moments, and many estimators involve division by random variables that have nonzero probability densities at value zero. Small sample properties tend to be mathematically intractable and thus are often investigated experimentally, though some theoretical results have been obtained. Nagar [1959] examines the small sample properties of Theil's k-class estimators in terms of the approximate distributions (that is the population distributions which are inferred from sample distributions) assuming that k is a function of n (that is $k = 1 + c/n$ where c is a constant).

It is shown that the k-class estimator is unbiased to order $1/n$ if $k = 1 + (h - h^* - g)/n$, so for example the TSLS estimator (with $k = 1$) is unbiased if $h - h^* = g^*$, that is if the equation has exactly one overidentifying restriction. A (approximate) matrix of squares and cross products of errors to the order $1/n^2$ is also obtained, together with an expression for c that is optimal in the sense that it minimises the determinant of this matrix; it is thus shown that this is likely to be negative (inferring that k should be less than unity) unless h is very large, which is not consistent with TSLS or LIML.

Basmann [1961] adopts a more particular approach; in the first case an equation

$$y_1 = \beta_{12}y_2 + \gamma_{13}x_3 + \gamma_{14}x_4 + \gamma_{10} + u_1$$

is postulated in a two equation model with four predetermined variables. The exact finite sample probability distribution of the TSLS estimator b_{12} of β_{12} is derived, and it is shown that this has a finite mean but infinite variance even when the true value is zero. In the second case it is assumed that x_3 does not appear at all in the model, where it is shown that the distribution of b_{12} has a finite mean and variance, and that if β_{12} is zero, then this mean is also zero; these results apply even if it is assumed that x_2 does not appear, so the equation is not identified.

These studies however are of limited practical help here in that Nagar's results are approximate (in the sense described above) and Basmann's are particular. There are also a number of empirical studies which may help in the evaluation of different methods, though these too are inevitably particular. These (Monte Carlo) studies are simulation studies: a model is postulated with known parameters (including a known distribution function of the residuals), and a set of data is generated to be consistent with these parameters (and distribution of the residuals), then the required parameters are estimated by the methods being investigated. The estimation process is repeated a large number of times using different subsets of the generated data, thus generating probability distributions of estimates of the parameters under different estimation procedures. There are three conventional measures for interpreting the results of such experiments: bias, $E(b) - \beta$; variance, $E(b - E(b))^2$; and mean-square-error (MSE), $E(b - \beta)^2$. The first is bias in the usual sense, the second is the variance of the estimates around their mean, and the third is the variance of the estimates around the true value - which is equivalent to variance (around the mean) plus the square of the bias. There are a number of such studies and we shall not attempt to examine all of these in depth, but merely to indicate the aims and general conclusions of some of the more relevant.

Basmann (in an unpublished paper summarised by Johnston [1963]) examines five parameters in one equation of a three equation model, with a sample size of 16 and the usual desirable properties, including normality; on the whole there is little difference between the performances of OLS and TSLS, but both are greatly superior to LIML. Wagner [1958] examines an over-identified two parameter consumption function in two similar three equation models with a sample size of 20; the bias of OLS appears slightly higher than that of LIML, but this is approximately offset in MSE terms by the lower variance. These two models are extended by Nagar [1960] to include TSLS and other k-class estimators, where again there is little difference between methods in the consumption equation, though there are real differences in another (the investment) equation: in MSE terms OLS is superior to TSLS in one model and TSLS to OLS in the other, though TSLS gives the lowest bias in both. Summers [1965] conducts several experiments with a two equation (supply and demand type) model with sample sizes of 20 and 40; the interesting cases where misspecification of the model and significant multicollinearity are present are also examined. In general FIML is superior, LIML and TSLS approximately equivalent, and OLS worst in the original model, but when misspecification and multicollinearity are present both FIML and LIML become inferior to TSLS. Neiswanger and Yancey [1959] examine a two equation model with a sample size of 25, particularly the effects of misspecification connected with the existence of time trends. When there is no time trend LIML is preferable to OLS, but when the data includes a time trend they are both similar and not very satisfactory; when time is included in the specification as a predetermined variable LIML again becomes superior to OLS. Ladd [1956] uses the same model, with a sample size of 30, and superimposes errors of measurement; these lead to little bias in either (OLS or LIML) estimator, but increase the variances. Quandt (in an unpublished paper summarised in Christ [1966]) examines two four equation models with a sample size of 20 with varying degress of multicollinearity among the predetermined variables, and finds that TSLS is preferable to OLS until the degree of multicollinearity becomes very high, when OLS is preferred. Quandt also computes estimates for a series of 25 k-class estimators with k ranging from -0·4 to 2·0, and obtains the important result that the estimator is relatively insensitive to the value of k (the mean estimate over 100 samples is fairly constant) for values from -0·4 to around 1·0 and from around 1·3 to 2·0, but in the range in the middle the mean estimate fluctuates violently. This conclusion, that the LIML estimator may be highly unstable if k is in the region of 1·1 to 1·2, is reinforced by Theil [1961], who estimates parameters by k-class methods for various values of k from real data (the model of Girshick and Haavelmo [1947] - thus this is not a Monte Carlo experiment), and notices the same phenomenon.

We now turn to some more general and practical observations. In practice the properties of estimators must be considered in conjunction with their robustness, both as regards misspecification and as regards the failure to (approximately) satisfy the assumptions on which the theoretical properties are based: particularly the absence of multicollinearity and serial correlation. It is generally recognised that FIML seriously lacks robustness; as regards multicollinearity Klein and Nakamura [1962] have suggested high sensitivity, and as regards misspecification it is clear that as the estimation of any equation depends on the specification of the entire model, any error in specification will tend to affect the estimates of all the parameters in the model. This is of particular importance in this model, where certain equations are of relatively unproven validity. At the practical level the complexity of FIML makes computation difficult, though with efficient programming of electronic computers not impossible for small systems (especially if more simple asymptotically equivalent forms are considered) though for a large model such as this the limitation of store space might prove to be an insuperable problem.

The fundamental difference between the two limited information methods being considered is that though both are members of the k-class of estimators the value of k in LIML is stochastic, being the root of a stochastic determinental equation, whereas the value of k in TSLS is deterministic, being in effect defined as unity. It is for this reason that LIML may be expected to be the less robust; Klein and Nakamura [1962] show that this is the case as regards multicollinearity, and it is intuitively apparent that this may well be the case in general. It may be recalled that TSLS requires the arbitrary normalisation of a dependent variable, whereas LIML treats all endogenous variables included in an equation. TSLS has been criticised on these grounds, but it may be argued, as it is by Fisher [1965], that this is a desirable rather than an undesirable facet of TSLS since normalisation rules are usually present in practice, as indeed they should be if the equation is based on formal theoretical grounds rather than an intuitive idea that certain variables are connected. These normalisation rules are thus in a real sense a part of the specification of the model, so the model is not completely specified unless each endogenous variable appears (at least implicitly) as a dependent variable. This is closely connected with Frisch's concept of autonomy, as incorporated in Tinbergen's [1939] criterion that relationships of a model should be as far as possible directly causal, for this implies an explicit dependent variable.

Computationally the limited information methods are relatively straightforward, though of the two considered TSLS is the simpler since LIML is complicated by the iterative procedure required to find the smallest root m. As knowledge of the complete structure and the observations on endogenous variables excluded from the equation being estimated are not required, computation is unlikely to pose any store problems.

OLS estimators are robust in the sense that their properties are not greatly affected by the type of phenomena being considered, though as these properties are not on the whole desirable for simultaneous systems this is of limited relevance. OLS is however the most simple method computationally, and thus if its other properties are not too undesirable may be useful for preliminary experiments.

Alternatives Before deciding on a method for estimating the parameters of a simultaneous structure it is relevant to examine whether the structural parameters are needed at all (it is possible that the reduced form parameters might suffice), and whether a simultaneous system is required.

If only the reduced form parameters of the model were required OLS might provide a suitable method, for the difficulties associated with OLS when endogenous variables are treated as independent now disappear. However if lagged endogenous variables are included among the predetermined variables then OLS is in general still biased, and even in the cases where it is unbiased is less efficient (asymptotically) than estimates of the reduced form derived from consistent estimates of the structural parameters. Thus it would appear to be preferable to estimate the structural parameters first by some simultaneous method even when only the reduced form is required - as in forecasting; indeed Christ [1966] suggests that this may be true even where OLS estimators of the structure are better than simultaneous estimates of the structure. The fundamental point for this model is that as its aim is the understanding of the underlying forces it requires a knowledge of the structure, not merely of the reduced form.

The model proposed is essentially simultaneous, and illustrates to some extent the interdependence of the economic system. Wold, [1953] and elsewhere, however has argued that this is a misrepresentation of economic forces, which are more validly interpreted as forming a causal chain, so that one variable only affects other variables in a stepwise and unilateral fashion. Such a system is termed recursive, and may be represented in the form

$$\begin{aligned}
y_1 \qquad\qquad\qquad\qquad &= z_1 + u_1 \\
\beta_{21}y_1 + y_2 \qquad\qquad &= z_2 + u_2 \\
&\;\;\vdots \\
\beta_{g1}y_1 + \beta_{g2}y_2 + \ldots + y_g &= z_g + u_g,
\end{aligned}$$

where the z's are linear combinations of the x's, and where no current disturbance is correlated with any past disturbance and the variance-covariance matrix of the current disturbances is diagonal. The equations of such a system may be estimated seriatim by OLS, for OLS will produce consistent estimates where there are no independent endogenous variables and these may be eliminated consecutively. If such a system were applicable consistent full information estimates of the parameters could readily be obtained by OLS, and so it is relevant to enquire whether the assumptions made by such a system are likely to be fulfilled. The first assumption is that the coefficient matrix $\underline{B}$ is triangular (in the way illustrated), which implies that all causation takes place sequentially in time. If we consider arbitrary small finite time periods then this is true, being implicit in the usual interpretation of the term 'causality', but in practice data almost invariably consists of averages over 'long' periods, when the model is in effect a simultaneous approximation to an underlying recursive model; in which case it should be estimated as fits the form it is, rather than the one it approximates. The second assumption, that there is no serial correlation in the disturbances, is unlikely to be satisfied in practice. This applies especially to short time periods, when the diagonality of $\underline{B}$ is most likely, for then a shock engendered by a variable omitted from the equation (and part of the justification for a disturbance term is that not all relevant variables can be included in practice) is likely to persist for more than one time period. Similarly the assumption that the variance-covariance matrix is diagonal, or that there is no correlation between disturbances in different equations, is unlikely to be satisfied, for omitted variables or random shocks are likely to affect more than one included variable, and thus in general the residual in more than one equation. Thus as a recursive system requires the satisfaction of three unlikely assumptions to be valid a simultaneous system is retained, though it may be recalled that this is an approximation to the underlying causal chains. Fisher [1965] has made use of a block recursive system, which is similar to a fully recursive system but requires the diagonality of a matrix whose elements are matrices of the parameters of the endogenous variables in parts of the system. This implies that the model is divisible into blocks, which though involving simultaneous relations between their constituent variables, are themselves only recursive. Since such a system still requires the choice of a method for estimating the parameters in each block simultaneously, and since it would not be logically plausible to divide this model into blocks other than by industry (and because of the nature of the links between industries such blocks could not be arranged recursively), this compromise is not pursued.

Method adopted It now becomes necessary to choose an estimation method. To summarise the two extreme methods, FIML seriously lacks robustness and is awkward computationally for large systems, while OLS is inconsistent and also apparently lacks any redeeming small sample properties; both are therefore discarded, leaving the choice (among the methods being considered) between LIML and TSLS. Many models are estimated by one of these methods, usually LIML, at first, then various equations are estimated by the other method if they do not appear acceptable. Such a procedure is open to serious criticism methodologically for it uses a priori considerations to reject an estimated equation, and assumes that the deficiency must be in the estimation method rather than in the specification (though more rational discrimination on the basis of the value of the smallest root m is possible). Conversely it will accept an estimated equation if it conforms with (possibly false) a priori ideas, when this may have been caused by an incorrect specification combined with one of the presumed deficiencies in the estimation method. If an estimation method is to be relied on if it produces results which conform to a priori ideas then it should be relied on when it does not; if it is not relied on then a more acceptable method should be sought. It is then necessary to choose between LIML and TSLS. The choice is based on the various facets discussed above, the deciding factor perhaps being the instability of LIML when m is in a (realistic) critical range, and the associated point of its being a k-class estimator with a stochastic k - an acceptable property usually but not consistently; thus TSLS is chosen. It is used for all estimates, though where an equation is just identified this would be equivalent to LIML, and where it is already in its reduced form it is equivalent to OLS. Finally the deficiencies of OLS may not be too great to prevent its use in preliminary experiments, where it is adopted; this is particularly valuable where all the predetermined variables are not known.

We now examine the main stages of the method of TSLS in such detail only as is necessary to derive an efficient computation procedure. We only examine the estimation of the first equation of the model, but as the ordering is arbitrary this involves no loss of generality. If the model is

$$\underline{By}_t + \underline{Gx}_t = \underline{u}_t$$

the first equation may be written as

$$\beta_{11}y_{1t} + \cdot \cdot \cdot + \beta_{1g*}y_{g*t} + \gamma_{11}x_{1t} + \cdot \cdot \cdot + \gamma_{1h*}x_{h*t} = u_t,$$

and this may be normalised in terms of y_1 as

$$y_{1t} = - \beta_{12}y_{2t} - \cdot \cdot \cdot \cdot - \beta_{1g*}y_{g*t} - \gamma_{11}x_{1t} - \cdot \cdot \cdot \cdot - \gamma_{1h*}x_{h*t} + u_t$$

where the coefficients represented by the β's and γ's are the new normalised parameters, not equal to the β's and γ's in the first equation.[3] We further define $\underline{Y}^*$ as the n x (g* - 1) matrix of observations on the explanatory endogenous variables in the equation, so that $\underline{Y} = \left[\underline{y}_1 \vdots \underline{Y}^*\right]$, and $\underline{X}^*$ as the n x h* matrix of observations on the predetermined variables in the equation, with the associated parameter vectors $\underline{\beta}^*$ and γ^* defined similarly. The normalised equation may now be written as

$$\underline{y}_1 = - \underline{Y}^*\underline{\beta}^{*\prime} - \underline{X}^*\underline{\gamma}^{*\prime} + \underline{u}_1.$$

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3. For typographical simplicity they are not modified; no confusion should arise as the two notations are not mixed - further, the only difference is the constant factor β_{11}.

TSLS avoids the difficulties brought about by the correlation between $\underline{u}_1$ and the variables in $\underline{Y}^*$ by replacing $\underline{Y}^*$ by its estimate as given by its OLS regression on all the predetermined variables, $\underline{X}$. Thus each of the variables in $\underline{Y}^*$ are regressed on $\underline{X}$, giving (from the basic OLS result) the estimate of $\underline{Y}^*$ as

$$\text{est}(\underline{Y}^*) = \underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Y}^*.$$

Now replacing $\underline{Y}^*$ by est$(\underline{Y}^*)$ in the original (normalised) equation gives

$$\underline{y}_1 = - \underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Y}^*\beta^{*\prime} - \underline{X}^*\gamma^{*\prime} + \underline{u}_1,$$

or $$\underline{y}_1 = - \left[\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Y}^* \;\vdots\; \underline{X}^*\right] \left[\beta^* \;\vdots\; \underline{\gamma}^*\right] + \underline{u}_1;$$

applying OLS to this equation, again using the OLS result, gives the TSLS estimates of the parameters as

$$\left[\underline{b}^* \;\vdots\; \underline{g}^*\right] = - (\underline{Z}'\underline{Z})^{-1}\underline{Z}'\underline{y}_1$$

where $$\underline{Z} = \left[\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Y}^* \;\vdots\; \underline{X}^*\right].$$

The OLS parameter estimates are of course obtained by replacing $\underline{Z}$ by $\left[\underline{Y}^* \;\vdots\; \underline{X}^*\right]$.

The error of estimate, $\underline{e}$, is defined as the difference between the estimated and true values of the parameters, so

$$\underline{e}_1 = \left[\underline{b}^* \;\vdots\; \underline{g}^*\right] - \left[\underline{\beta}^* \;\vdots\; \underline{\gamma}^*\right],$$

and by using the result above this may be written as

$$\underline{e}_1 = - (\underline{Z}'\underline{Z})^{-1}\underline{Z}'\underline{u}_1.$$

The asymptotic variance-covariance matrix for the estimators is then defined as the limit as n tends to infinity of

$E(n\underline{e}_1\underline{e}_1')$, and this may be shown to be equal to $s^2(\underline{Z}'\underline{Z})^{-1}$ where s^2 is the variance of $\underline{u}_1$. This is usually taken as the sum of squares, $\underline{u}_1'\underline{u}_1$, divided by the relevant degrees of freedom, $n - g^* - h^* + 1$, though Nagar [1961] has shown that division by n may be more appropriate. The former (smaller) division is however used as this will produce safer (larger) estimates of the variances. The (asymptotic) standard error of a parameter is readily derived from the principal diagonal of this matrix, being the square root of the relevant diagonal element multiplied by s.

The purpose of the above discussion is solely to illustrate the main stages of the method in order to derive some efficient computational procedure, and thus the results are not in their most usual form. A full treatment is not given as this may be found in many sources originating from Theil [1961]. Proofs that the estimates exist (that is that $\underline{Z}'\underline{Z}$ is nonsingular) if and only if the equation is identified, that the limit as n tends to infinity of $E(n\underline{e}_1\underline{e}_1')$ is $s^2(Z'Z)^{-1}$, and that the estimators have the symptotic properties of consistency, normality, and efficiency are also proposed by Theil, but are given in more general forms by Christ [1966].

Appraisal of estimates The appraisal of the estimates of the parameters of a model (as opposed to the appraisal of the whole model) is based on their actual values, and on the variances of these. The estimated values of the parameters should accord with a priori reasoning, and also with any available external empirical information. The exact variances of the parameter estimates are not however known, only the approximate (asymptotic) variances, but these may be considered as the best estimates of the true variances and used to evaluate the confidence placed on the estimates of individual parameters in the standard way (using the t or F tests), as long as their approximate nature is recalled.

Confidence in the process whereby the estimates and their variances are derived will depend on the absence of serious multicollinearity and serial correlation of the disturbances. Both may be tested for; the former from an examination of the simple and multiple correlation coefficients among the predetermined variables, and the latter from the standard von Neumann ratio calculated from the observed residuals.

The overall goodness of fit may be measured by the standard error of estimate, that is the square root of the variance of the observed residuals, though this measure clearly depends on the size of the dependent variable; it is thus useful for evaluating one form of an equation relative to another form, but not absolutely. Deflating this by the mean of the dependent variable is of little use if this may be near zero, and deflating by the variance merely produces a figure analagous to the multiple correlation coefficient, R^2, in the single equation case (in fact $1 - R^2$). Thus if an absolute measure is required R^2 may be useful in the single equation case, though its meaning is not clear for a simultaneous model; indeed it is 'of no value as an indicator of the usefulness of a structural equation' (Christ [1966, p. 519]). Basmann [1962], in drawing attention to this misuse, defines R^2 as 1 - (residual variance)/(total variance), and shows that it is misleading in that it may well be low (or even negative) without compromising the degree of confidence to be placed in the equation. This is illustrated by postulating a two equation supply and demand cobweb model, and showing that if the supply function is inelastic and has a residual with a low variance (relative to the demand function) then the measure of R^2 may well be negative. It is further shown that even for evaluating a reduced form equation this statistic is severely compromised by the fact that its probability distribution depends in a complex manner on the structural coefficients.

Basmann however only focuses on half of the problem, for in the single equation case R^2 may be equivalently defined as (explained variance)/(total variance). Writing the model as

$$y_t = x_t + u_t$$

where x_t is a composite of all explanatory variables, this is because

$$\text{var}(y) = \text{var}(x) + \text{var}(u)$$

or

$$1 - \frac{\text{var}(u)}{\text{var}(y)} = \frac{\text{var}(x)}{\text{var}(y)}.$$

In general however

$$\text{var}(y) = \text{var}(x) + \text{var}(u) + 2\text{cov}(x, u)$$

and cov(x, u) is only zero in the single equation model, that is where no endogenous variables (correlated with u) are included in x. Thus in general the two measures of R^2 are not equivalent; we may term the former (used by Basmann) the alienation coefficient, R_a^2, and the latter the correlation coefficient, R_c^2, following Hotelling's [1936] vector terminology. It is clear that

$$R_a^2 - R_c^2 = 2\mathrm{cov}(x, u)/\mathrm{var}(y),$$

that is the difference between the two measures is twice the covariance divided by the total variance. This suggests the possibility of a compromise measure, R_b^2, which includes the covariance divided by the total variance only once, that is

$$R_b^2 = R_a^2 - \mathrm{cov}(x, u)/\mathrm{var}(y) = R_c^2 + \mathrm{cov}(x, u)/\mathrm{var}(y)$$

or

$$R_b^2 = 1 - (\mathrm{var}(u) + \mathrm{cov}(x, u))/\mathrm{var}(y) = (\mathrm{var}(x) + \mathrm{cov}(x, u))/\mathrm{var}(y).$$

R_b^2 clearly is the mean of R_a^2 and R_c^2, so the three measures coincide when the covariance is zero. It should be emphasised that this compromise measure is the average of two deficient measures, and thus can only be regarded as a crude approximation to some more meaningful figure such as Hooper's (see below); in particular it may be noted that this measure may be negative or exceed unity, though such occurrences would appear to be very rare. It is however used here as there is no simple alternative and it may well be of (limited) use if its deficiencies are recognised. Hooper [1959] develops a measure of the proportion of the total variance of the endogenous variables as a group that is explained by the predetermined variables as a group; this trace correlation coefficient may be regarded as a matrix generalisation of the scalar R^2 - in the degenerate one equation case it may be shown to be equal to R^2. This is not however relevant to the evaluation of individual equations and is computationally complex, particularly as regards use of store space. Hooper's [1962] partial trace correlation coefficient could ideally be useful, but again is too unwieldy for practical use here.

Practical aspects The most immediate practical problem is that of insufficient degrees of freedom in the reduced form, for there are only thirtysix observations but a substantially larger number of predetermined variables (mainly due to non-linearities and lagged values), so it is not possible to estimate the full reduced form parameters. Even if sufficient observations were available such estimation would be made very difficult because of multicollinearity, for it is improbable that a large number of (even predetermined) variables would not contain at least one highly collinear set. Thus estimation is by blocks - the model being divided into sub-models each of which is estimated by TSLS using a subset of the available predetermined variables. It is apparent that there will be decreasing returns for each predetermined variable added, for the addition of a variable will in general increase the degree of multicollinearity somewhere so the new variable adds little information; thus it may be reasonable to include only twenty or so predetermined variables. A block may consist of one equation, but such fineness is both time consuming and unnecessary. In practice blocks should consist of relatively highly interconnected equations; these may be chosen by inspection, which necessarily introduces a subjective element, or by an analysis of the causal structure as suggested by Fisher [1965] - a process which may also prove valuable when the estimated model is to be solved. After the system has been divided into blocks a set of predetermined variables is chosen for each block. Clearly those appearing in the block must be included, but there may be some choice as regards others; here again choice may be subjective, though more rigorous methods are available: Kloek and Mennes [1960] suggest a procedure based on the principal components of predetermined variables, and Fisher develops some rules from a study of the causal ordering of the system. These methods are not examined here as although this model is large it differs from other large models in that it is clearly divided into industry blocks, and so the problem of division into blocks does not arise more than formally. Blocks then are industries, and the number of predetermined variables in each industry model is approximately the number required for each block, so these (all the predetermined variables in the outline industry model - for each industry separately) are the only predetermined variables taken account of.

Finally there is the computational aspect. Computation was performed entirely by electronic computer for which a program (basically for TSLS but incorporating OLS as well) was specially written; a copy of its published specification is given in appendix C.

It is difficult to comment on the numerical accuracy of the estimates produced for hand computation of even one equation to a reasonable number of significant figures is out of the question. The degree of accuracy is however of particular relevance where there is a significant degree of collinearity between variables (as there is here between some predetermined variables) since matrix inversion then involves division by the difference between two very similar figures. The problems have been investigated by Longley [1967], who regresses employment in seven sectors, and total employment, on a set of six highly collinear independent variables, both by hand to eight significant figures and using a number of standard computer programs. The results are interesting: in many cases even the first significant figure of the computer estimated coefficients is incorrect, and some even have the wrong sign. A further test is provided by the aggregate regression, where the coefficients should be the sums of the individual coefficients, a condition which is rarely satisfied by the computer estimates. The main reason for the inaccuracy of most of these computer estimates is that the computers tend to have short word lengths (sufficient to carry around eight digits), though a correspondingly large number of words in the store. The machine used for this study (KDF 9) however has an unusually large word length (sufficient to carry around eleven digits), and a small store; this makes estimation awkward yet more accurate. Longley's model (with appropriate scaling) was estimated by the OLS variant of the program written, with encouraging results: all the parameter estimates were found to be correct to the number of significant places given in appendix D (that is three), and the aggregate coefficients were found to be the sums of their respective components. This provides no direct indication of the accuracy of the TSLS estimates, but is encouraging, for the TSLS program uses the same routines for matrix inversion (which is where inaccuracy is most likely to originate) and the same machine as the OLS version; the matrices are however larger in the TSLS computations, and so are more likely to contain collinear variables. Thus the possibility of numerical inaccuracy, though perhaps smaller than in many studies, does exist.

4-2 Industry estimates

This section discusses the numerical estimates of the industry models that are presented in appendix D; it is an assessment and interpretation of the figures, emphasising some interesting results, and is thus in no way a substitute for the appendix - indeed the discussion should be interpreted in conjunction with the appendix. A rigorous analysis of the implications of all the particular findings is regrettably beyond the scope of this study.

The estimates presented are the results of estimating the theoretical outline industry model by TSLS for each of the ten industries, and making various modifications to this outline model in certain industries in the light of empirical evidence. In the discussion of the particular methodology of the model in chapter 1 it was stressed that the function of the outline model is to express as far as possible the relations that apply to any industry, whatever the nature of its technology or state of competition, and that the main difference between industries would be the varying importance of certain factors, reflected in the different (possibly zero) values of the estimated coefficients. Thus little basic modification is necessary, and where this does occur the underlying reasons are readily apparent. In the interests of simplicity variables are removed from relationships if their coefficients are of virtually no statistical significance (this particularly aids clarity where they are, insignificantly, in a theoretically impossible range); the lowest level of significance of included variables is that where the coefficient is one third of its standard error (in modulus) - a very liberal interpretation of significance.

The estimates of individual parameters may be assessed according to their actual values and their t ratios; the complete equations may be assessed according to their standard errors of estimate, von Neumann ratio, and tentatively the goodness of fit - the statistic approximately analagous to the multiple correlation coefficient in the single equation case. The theoretical interpretation of these is standard or has been discussed above; it should however be noted that the goodness of fit statistic includes (in effect) the proportion of the variance explained by the seasonal terms. In numerical terms the value of t required for significance at the five percent level is approximately 2·0 (its exact value depends on the number of variables in the equation), and the value of the von Neumann ratio indicating no serial correlation of residuals is approximately 2·0; positive autocorrelation may be inferred from values less than 1·5 (again approximately, and at the five percent level of significance), and negative autocorrelation from values greater than 2·5. Outlying observations are those time periods for which the residual is of exceptionally large absolute size, such as would be expected to occur in only one percent of observations (assuming that the residuals are normally distributed), that is greater than 2·75 times the standard error of estimate. No mention is made of the seasonal or constant terms.

The individual stochastic equations are now discussed in the order in which they were presented in chapter 3.

1. Demand As it was indicated in chapter 3 that this equation is virtually an identity the estimates of its parameters require little discussion; as is to be expected the estimates appear very satisfactory. The intermediate demand variable is very significant, and appears to account for a reasonable proportion of total demand, this being lowest for industry n; it should be recalled however that this is defined in part by the dependent variable. It is clearly worthwhile including changes in stocks, for these are relevant for all industries but one (e), usually with reasonable significance; this is the only element of demand that is directly determined by the industry, though the industry's production decision may be influenced from within through its pricing decision. The final demand components are of reasonable importance, and are in general associated with the industries expected. There is only slight evidence of (positive) serial correlation of the residuals, and as is to be expected the goodness of fit statistic is everywhere extremely high. Two industries (p and o) each have one (the same) outlying observation.

2. Supply This is essentially a linear production function which is used as a demand for labour function; the coefficients of both factors should of course be positive. The theoretical form proposed appears satisfactory, for with one exception both labour and adjusted capital are relevant for all industries, usually (the latter always) with reasonable significance. The overall goodness of fit is high, and there are no outlying observations; some serial correlation of residuals is however present in most industries.

A linear function can only indicate approximate figures for marginal productivities. These are best interpreted from the average elasticities of output, since these are independent of the units used and the units of output (index numbers) in different industries are not immediately comparable. These average elasticities, the average percentage change in output associated with a unit percentage change in the factor input, are calculated by multiplying the relevant coefficients by the ratio of the mean of the factor series to that of the output series; they are given in table 4-1. As was indicated in chapter 3, some implications of the main alternative formulation are also presented here - the production function being a particularly important equation. Thus table 4-1 also gives the marginal productivities of factors as derived from a modified form of the Cobb Douglas function. This function is modified by the incorporation of additive seasonal terms in the logarithmic form (or equivalently multiplicative exponential seasonal terms in the basic form), and by the lack of any constraint on the parameters of the two factors (particularly that their sum is not constrained to be unity); the modification is to facilitate comparison. This alternative form expressed in logarithms is thus

$$\log x = \alpha_1 \log l + \alpha_2 \log k_{-1} l/(l + u) + \omega.$$

For the preliminary experiments mentioned in chapter 3 this was estimated by OLS; here it is re-estimated by TSLS, again to aid comparison.

On the whole these elasticities indicate greatly varying degress of economies of scale in different industries, but do not appear implausible given the approximate nature of the figures. The one exceptional figure is that for capital in industry t, which appears very large; this is perhaps understandable in view of the nature of the industry - with rationalisation against a background of high excess capacity producing an increase in output (for a given labour force) with little addition to capital stock. In general the marginal productivities or output elasticities implied by the two forms of the equation correspond very closely. The largest difference occurs for labour in industry e; there is no immediately obvious explanation for this, for employment in this industry is not particularly volatile.

The one industry where both factors are not relevant is industry c, where labour appears completely insignificant. This is not a particularly surprising result considering the extreme capital intensiveness of this industry, and indeed an adequate production function is obtained by omitting labour,

Table 4-1: Output elasticities

Industry	Labour		Capital	
	Linear	Cobb Douglas	Linear	Cobb Douglas
n	1·15	1·31	1·54	1·50
f	0·38	0·33	0·72	0·73
c	-	0·04	1·11	1·10
i	1·81	1·87	0·23	0·21
e	0·28	0·44	1·08	0·99
v	0·96	0·96	0·80	0·80
a	0·29	0·26	0·18	0·17
t	0·46	0·47	3·27	3·33
p	0·31	0·46	1·18	1·11
o	1·52	1·71	0·44	0·40

for capital remains very significant, producing a high goodness of fit, though serious serial correlation of residuals. It is however clearly inadequate as a labour demand function, and this leads to the first of the modifications to the outline model. The zero effect of output on the demand for labour is interpreted as the demand for labour being purely institutionally determined, or as all labour being overhead labour; this is formalised in the simple model

$$l = l_{-1} + \text{(residual)},$$

that is that the demand for labour is last period's demand plus some purely random residual. Alternative relationships were considered, of which perhaps the most relevant was that more labour is not needed to increase output from a given amount of plant, but that more plant requires more (overhead) labour, that is

$$l = \alpha + \beta k + \text{(residual)},$$

where α and β are parameters. Experiments however indicate that the former model is as satisfactory, and so being simpler is adopted. This equation, which requires no estimation, thus replaces the outline supply equation in industry c; this leads to further necessary changes for this industry.

3. Stocks This equation combines an expectational accelerator with speculative motives; all coefficients except that of bank rate should be positive, and that of the last period's change of stocks should be less than unity. The accelerator effect appears relevant in six industries, but price speculation appears somewhat more important, being relevant in all but two industries (n, understandably in view of the price series used, and i); the cost of speculation is however only of minor importance, bank rate being relevant for only three industries, and then with little significance. The expectational approach would appear to be valuable, the lagged change of stocks term being relevant for all industries but one (f), usually with reasonable significance. The goodness of fit is acceptable in view of the extremely volatile nature of the dependent variable, and except in industry f (which does not include the lagged change of stocks term) there is no evidence of any serial correlation of residuals. Three industries each have one (different) outlying observation.

The estimates of this equation suggest that the speculative effect is slightly more important than the acceleration effect, though this conclusion is tentative. This is contrary to most findings, for example those of Lovell [1964], which suggest that firms do not speculate in stocks - or at least not in their total volume or not in accordance with recent price behaviour. Although lending some support to the speculative effect the estimates presented here do not remove support from the accelerator hypothesis, for this is generally taken to relate desired stocks to anticipated sales, neither of which are observable. This model has attempted to allow for these unobservable values according to certain subsidiary hypotheses, but as these hypotheses are by their nature untestable no firm contradiction of the basic accelerator hypothesis may be inferred, and only a certain doubt is thrown on this formulation; it may however be noted that unless some hypothesis is made about these unobservable values the basic accelerator theory is not, in the Hicksian sense, meaningful.

It may be noted that the acceleration coefficients are all very small, thus it would appear that even when the accelerator is of absolute significance it is of relatively small importance. The acceleration coefficient may be interpreted as the value of the change in stocks associated with a unit change in the value of sales over a unit time period; the dimension is thus that of time, and the coefficient could be interpreted as a crude indication of the 'payback period' if all stock formation were associated with the accelerator. The acceleration coefficient (in units of quarters of a year) is thus obtained by dividing the coefficient of the change in the index by the base of the index for the relevant industry (both from appendix D);

these values are presented in table 4-2. For comparison the fixed investment accelerators (brought to the same units) are also presented in this table.

Table 4-2: Acceleration coefficients (quarters)

Industry	Stocks	Fixed investment
n	0·44	-
f	1·23	-
c	-	-
i	0·35	0·13
e	0·41	0·08
v	0·06	-
a	0·31	-
t	-	0·21
p	-	-
o	-	0·10

It should be recalled that some of these figures are open to a fairly wide margin of error (their t ratios are simply those in the appendix), but they do at least suggest that the accelerator is of marginal importance here. (The fixed and inventory investment accelerators are compared in the discussion of equation 5.)

4. Labour This equation expresses the supply of labour in terms of its relative remuneration and general availability; all coefficients should be positive, and those of aggregate labour and unemployment should sum to less than unity. Relative remuneration does not appear to be of great importance; it is relevant for all but three industries, but frequently with little significance. The two measures of general availability are important, for with one exception aggregate

labour is significant in all industries, and aggregate unemployment in half of the industries. The exception is industry t, which has uniquely experienced a steady decline in labour force over the period, and for which the availability of labour is clearly of little relevance. The overall performance of this equation is only moderate; goodness of fit is usually satisfactory, sometimes high, but all industries have significant positive serial correlation of residuals. There are two industries (e and p) with one (the same) outlying observation.

Again industry c needs revision, for if labour demand is determined institutionally the supply of labour may be expected to contract at times when labour demand is relatively low and the expected value (in the statistical sense) of earnings for each member of the labour force is also low - though the expected value for those in employment remains high. Thus where employment or unemployment appears arbitrary (or is not determined through output) and does not directly affect the level of earnings (see equation 7 below), the level of relative earnings can only be taken as an indication of the attraction of the industry if it is weighted in some way to allow for the probability of those earnings not being received, that is of the worker being unemployed. This is clearly the case for industry c, for when no account is taken of this none of the three explanatory variables of the outline model appear relevant, yet when an additional variable $1/u$ is included to take account of this they all become relevant and $1/u$ is itself highly significant. This is the first of two changes to the basic industry model that requires re-estimation; the basic and modified estimated equations are presented here. The figures in parentheses under the coefficients are their t ratios, VN is the von Neumann ratio, GF is the goodness of fit, and w is a composite of the constant and seasonal terms; the modified equation is given here for ease of comparison - it is also given in appendix D. The equations are

$$\text{basic:}\quad l + u = \underset{(0\cdot4)}{-19\cdot5}\ e/E + \underset{(0\cdot0)}{0\cdot000237}\ L - \underset{(1\cdot4)}{0\cdot0409}\ U + w; \quad \begin{array}{ll} \text{VN} & 0\cdot4 \\ \text{GF} & \cdot14 \end{array}$$

$$\text{modified:}\quad l + u = \underset{(1\cdot0)}{48\cdot4}\ e/E + \underset{(2\cdot7)}{0\cdot0204}\ L + \underset{(2\cdot5)}{0\cdot174}\ U + \underset{(3\cdot2)}{0\cdot854}\ l/u + w; \quad \begin{array}{ll} \text{VN} & 0\cdot8 \\ \text{GF} & \cdot44 \end{array}$$

Other ways of incorporating this effect are possible, perhaps the most direct being the replacement of e/E by its expected value, $(e/E).l/(l + u)$, but this assumes that the income of those not employed would be zero, which is clearly not the case if they receive unemployment compensation. Thus for industry <u>c</u> the labour equation at this stage is

$$l + u = \alpha_{41}e/E + \alpha_{42}L + \alpha_{43}U + \alpha_{44}l/u + \omega_4.$$

An examination of this equation in conjunction with the labour demand equation suggests an important further change in the final model. This arises because the labour demand equation essentially determines l, which is a large part (about 98 percent) of the dependent variable $l + u$ of the labour supply equation; thus a small inaccuracy in the solved values of l or $l + u$ would lead to a large inaccuracy in the solved value of u. For example if the true values of ł and ł + u were 99 and 101 and the solved values were 101 and 99 respectively (that is each involving about 2 percent error), the solved value of u would be -2 instead of +2, which is clearly unacceptable as the inverse of u is used elsewhere (in equation 7). This is unavoidable if u is expressed as the difference between ł + u and ł, which is the only theoretically meaningful way. The modification adopted places a small arbitrary lower bound to the value of u, which is taken as one percent of the value of ł. If j is defined as

$$j = \alpha_{41}e/E + \alpha_{42}L + \alpha_{43}U + \omega_4,$$

(with the additional term mentioned above for industry c), then the effect of this is to produce a discontinuous function for u,

$$u = f(j;\ \text{ł})$$

in the first quadrant such that f is at first a constant function taking the value ł/100, then linearly increasing. Thus the labour equation becomes

$$\text{ł} + u = \max[j,\ \text{ł} + \text{ł}/100]$$

for all industries where j is defined as above. It is not suggested that the true function is discontinuous; this function is only proposed as a bilinear approximation to a more complex asymptotic function. This (rather arbitrary) modification makes the equation more suitable for prediction in a simultaneous context.

5. Capital The fixed investment equation may be considered to be parallel to the inventory investment equation, determining investment by a combination of the expectational accelerator, profitability or liquidity, and desired stock or replacement motives. All coefficients except that of lagged capital stock (positive or negative) should be positive, and that of lagged investment should not exceed unity. The acceleration principle appears to be of minor importance, being at all relevant in only four industries and then with little significance. The profitability or liquidity effect appears to be the basic determinant, being relevant for all industries, usually significantly. The lagged capital term is relevant for all industries except one (p), but at slightly lower levels of significance; the coefficient is positive in five of the nine industries indicating that the depreciation or replacement effect outweighs the desired stock effect, the converse applying in the other four. Expectations are very important, for the lagged investment term is relevant in all industries except one (o), usually with very high significance. The goodness of fit is on the whole surprisingly high considering the nature of the dependent variable, and there is no evidence of serial correlation of residuals in any industry. Industry n has one outlying observation.

The implications of these estimates on the relatively small importance of the accelerator are contrary to a number of findings, for example those of Eisner [1964], and are thus surprising - particularly so since the equation appears on the whole to be satisfactory statistically. The remarks on the accelerator made above, that the basic relation concerns two unobservable variables and that the apparent failure of an observable extension may merely imply an incorrect interpretation of this, apply equally here. Thus it is possible that for example the wrong lag structures are being used, though as was mentioned in chapter 3 extensive experimentation with other lags yielded substantially similar results. A more fruitful explanation of the greater apparent importance of profits than the accelerator concerns the prima facie similarity of the two motives. As Eisner [1964, p. 173] shows, 'with almost any reasonable production function, one should expect increases in demand sooner or later to generate capital expenditures, and profits to be associated with capital expenditures only to the extent that they themselves were associated with the pressure of demand on capacity. Capital expenditures would be associated with profits per se only where imperfections of capital markets were likely to be significant'. The first part of this statement suggests that profits are in general relevant only as a proxy for other factors (notably capacity utilisation), and it is likely that some part of the profits variable included here is important only as a proxy for other factors connected with the acceleration motive which must for practical reasons be excluded. The latter part of the statement is however equally important, for imperfections in the capital market are widely recognised, and indeed form the basis of financial capitalism. Thus some part of the profits variable is likely to reflect the relevance of profits per se; unfortunately information from outside the scope of this study would be required to identify the sizes of these two parts. The immediate inference that the profits effect is more important than the accelerator effect may thus not be refuted, but may be qualified to be relevant only when serious imperfections in the capital market exist, which apparently constitutes the environment of this study.

The actual values of the acceleration coefficients are small, as are those for inventory investment (both are presented in table 4-2 above); again it appears that even when the accelerator is absolutely significant it is of relatively small importance. It would appear that the fixed investment accelerators are smaller on the whole than those for inventory investment, though there is not sufficient evidence to establish this more than very tentatively. This would however be a somewhat surprising result in view of the observable large differences in the total amounts of fixed and inventory capital required for the same output, though these amounts indicate the average rather than the marginal figures, which need not (despite our development of the concept) be assumed equal. The main inference that may be drawn from the relative and absolute sizes of these figures however is that the accelerator is of marginal importance here.

6. Price The price formation equation is based on a modified markup process, where the coefficients of the two cost terms should clearly be positive, and that of the level of stocks relative to output in the last period negative. As mentioned above this equation does not apply to industry n, where it is replaced by a definition which states that the price in this

industry is that for the economy, which is thus equivalent to the average of the prices of the other industries, or the price for m.

In all industries where the basic equation does apply the price of materials is of prime importance, being relevant for all industries with very high levels of significance; it should be recalled however that this is defined in part by the dependent variable. Unit labour costs appear to be of less importance, being relevant in five of the nine industries with only moderate significance, while the pressure of demand term is only relevant for four industries, with slightly less significance. With one exception the overall performance of the equation is good, with high goodness of fit and usually only small evidence of serial correlation of residuals; the exception is again industry c, where goodness of fit is fairly low and there is serious positive autocorrelation of residuals, but where no superior relation could be found. Industry e has one outlying observation.

The estimates of this equation are substantially as expected, and are not discussed further.

7. Earnings The earnings equation expresses the rate of change of earnings as a function of four separate factors, each of which should have a positive effect. Rather surprisingly unemployment does not appear to be very important, being relevant in only half of the industries, and then with no great significance. The rates of change of output and of profits are each relevant in four industries, at only moderate levels of significance, and for no industries are both relevant; this suggests that they are to some extent

substitutes, and in fact these variables are significantly collinear, despite their different time periods. The rate of change of consumer prices is relevant for half of the industries. Closer investigation shows that seasonal factors are of particular importance, possibly reflecting the annual nature of the wage component in earnings; ideally more of this seasonal pattern should be explained, but no better formulation could be found. This applies particularly to industry v, for which the equation is (perhaps not surprisingly) particularly poor, where the seasonal factors appear to be the only relevant factors; the equation for this industry is this purely artificial or empirical. This is clearly undesirable but is retained as extensive investigations showed no theoretically and statistically satisfactory alternative.

Industry c again forms a special case, for as is to be expected in the light of the discussion above on employment and unemployment in this industry, the equation incorporating industry unemployment is very inadequate, but becomes slightly more acceptable only when industry unemployment is replaced by aggregate unemployment; in both cases unemployment is the only relevant term, and the equation is not particularly satisfactory. This is the second of the two modifications to the basic model requiring re-estimation; the basic and modified estimated equations (in the same format as in the first case - equation 4) are

basic: $e/e_{-1} = \underset{(0\cdot0)}{-0\cdot112}/u + w;$ VN 3·0 GF ·21

modified: $e/e_{-1} = \underset{(0\cdot5)}{5\cdot96}/U + w;$ VN 3·0 GF ·23

On the whole this equation is acceptable in most industries; goodness of fit is satisfactory, and there is only slight evidence of (negative) serial correlation of residuals. Industry p has one outlying observation.

The implication of these estimates on the importance of unemployment is at first sight contradictory to most work on Phillips relations. The superiority of industry to aggregate unemployment, the use of the reciprocal of unemployment, and the choice of lag were discussed in chapter 3. The basic difference however between this equation and most others is the use of earnings rather than wages as the dependent variable, and the difference in results may be tentatively ascribed to this. This would be compatible with the hypothesis that wages are determined primarily by the level of unemployment while the excess of earnings over wages is determined by other factors, and that the variance of the latter is large enough to swamp the former. As figures for wage rates are not used in the model this hypothesis cannot be tested here, but would appear plausible; indeed the decision to omit wages depended in part on their arbitrary, and static, nature relative to earnings. This effect might be expected to be emphasised in a quarterly model where the level of wages might not change for three quarters (wage bargaining being predominantly an annual process), and particularly so in a disaggregated model where annual wage changes in individual industries cannot through aggregation produce a series with more movement from quarter to quarter. This explanation must however remain tentative in the absence of further information.

8b. Profits (b) Profits in this equation are determined by sales, which should have a positive coefficient, and by two other variables whose coefficients may be positive or negative. As expected, sales are significant in all industries; both unit labour costs and lagged sales are relevant in all industries but *f*. The overall goodness of fit is high, and there is little evidence of serial correlation of residuals; there are no outlying observations.

Being one of the two possible profit relations this equation is clearly important, and it is relevant that it appears adequate at this stage. The equation has certain implications concerning the degree of competition in the various industries, for it is recalled that a primarily competitive industry (C) is associated with a positive coefficient for unit labour costs and a significant negative lagged sales term, whereas a primarily monopolistic industry (M) is associated with a negative coefficient for unit labour costs and no (or an insignificant positive) lagged sales term. The industries may thus be classified in two ways; these classifications are presented in table 4-3.

Table 4-3: Degrees of competition

Industry	Unit labour costs	Lagged sales
n	C	C
f	C	M
c	C	C
i	M	M
e	C	C
v	-	C
a	M	M
t	C	C
p	M	M
o	M	C

These results are reasonably internally consistent, agreeing in seven industries and disagreeing in two (unit labour costs are not relevant either way for industry v and thus do not give it any classification); this classifies five industries as being primarily competitive, three primarily monopolistic, and two indeterminate. It is not however clear how accurate these classifications are for the calculation of concentration ratios or other independent measures is outside the scope of this study, and our industry disaggregation does not readily correspond with that used in other work on competitive structures.

Summary The overall impression derived from the estimates is that the theoretical outline industry model is, with certain exceptions, reasonably satisfactory; when these exceptions are allowed for by the changes suggested above it becomes acceptable for further testing by prediction. These changes fall into three categories. Firstly there is the change due to the lack of data, that is in the price equation in industry n. Secondly there are the more fundamental changes to the model for industry c which are occasioned by the unimportance of labour (at the margin) in this industry's production process; these are the replacement of the output related labour demand function by an institutional relation, the allowance for unemployment in the labour supply function, and the replacement of industry by aggregate unemployment in the earnings equation. Thirdly there is the modification of the labour equation in all industries to make it more suitable for prediction in a simultaneous equation context.

As part of the purpose of this study is the formulation of an outline model which would describe any industry the need for these changes is a prima facie criticism. The main purpose of the model however is to produce industry models accurate enough to be combined into a reasonable whole, and the changes made have acceptable theoretical bases: the first change is trivial, the second changes are all interrelated and are made necessary by the observed unimportance of labour in industry c, and the third applies an important logical constraint (albeit in a rather arbitrary way).

It is relevant that for the 78 (that is 10 x 8 - 2) estimated industry equations, each involving 36 observations, there are only 10 outlying observations (as would be expected in only one percent of the observations if the residuals were normally distributed), whereas approximately 28 could be expected. Further, five of these ten are for the same observation (1963 Q1); it did not seem worthwhile, and would have been of doubtful validity, to re-estimate the model without this observation. This suggests that the residuals are not normally distributed, but are supernormally kurtotic, which contradicts one of the ideal assumptions of the estimation process. The effect is however in an acceptable direction for the assumption of normality in such a case would tend to produce larger standard errors (of parameters and estimate) than the true values, thus greater confidence may be placed on the inferences from the values obtained, as the true values will tend to be smaller.

4-3 Economy estimates

This section discusses the numerical estimates of the industry models that are presented in appendix D. The comments introducing the last section apply here also, though this section is briefer partly because the specification of each equation estimated (not just an outline equation) is discussed in chapter 3, and partly because the emphasis of the study is on the industry models. The equations are discussed under the three headings under which they were presented in chapter 3.

Consumer sector Consumption is determined primarily by disposable labour income, this being of prime importance for all categories of consumption except that of motor vehicles. The short run marginal propensities to consume (with respect to disposable labour income not total income) are given by the coefficients of the term DLI, and the long run propensities, that is the propensities where consumers are no longer adjusting their expenditures, are given by reformulating the equations with current consumption equal to last period's consumption. These figures however have little theoretical meaning as property income may also be spent on consumption; the equations are only based on the concept of disposable labour income for empirical reasons, this being justified only by the concentration of this model on industry behaviour. Relative prices are important for all categories except, as might be expected, that of food, drink, and tobacco, and the expectational factor provided by the lagged consumption term is important for all categories except clothing and footwear. Hire purchase restrictions, introduced specifically for the motor vehicles category, are relevant, though not with particularly high significance. The overall goodness of fit is very high and there is no evidence of significant serial correlation of residuals.

Consumer prices are determined by a markup process from wholesale prices with allowance for purchase taxes. As is to be expected wholesale prices are always very important; tax rates are important for all categories except the residual one. The overall goodness of fit is very high except for the motor vehicle category, but there is definite serial correlation of residuals.

Foreign trade Both categories of exports depend very significantly on the level of world trade, with the expectational factor being relevant for nonmanufactures but not for manufactures. Goodness of fit is high and the residuals are reasonably serially independent.

Both categories of imports depend significantly on total factor income, both with significant distributed lag effects. Goodness of fit is high, with slight evidence of positive serial correlation of residuals.

Empirical relations There is little to be said about these in view of their pragmatic nature; both however fit very well and thus serve their purpose.

CHAPTER 5 SOLUTION AND ASSESSMENT

This chapter discusses the solution of the systems of equations which specify the model, and assesses the model as a whole; first absolutely in terms of its predictive ability, especially of profits, then as a plausible simulator of the interindustry tatonnement process. For reasons discussed in chapter 2 this study allocates nearly all of the available observations of the real world to the estimation rather than the testing of the model; accordingly this chapter seeks to present a tested methodology for the solution and assessment of the model with some indications of its general validity, rather than an exhaustive analysis of its properties. The solutions discussed are presented in appendix E.

5-1 Theory of assessment

Assessment of a model may be either predictive or nonpredictive. The latter is usually based on the sample period, when it depends on the use of statistical inference about the residuals in this period. If as is desirable, the model itself is derived from the concurrent development of theory and observation during the sample period, then the usual processes of statistical inference cease to apply and nonpredictive assessment is inappropriate - it will in general give too optimistic an assessment of the model for modification in the light of observation is modification to fit (not necessarily explicitly) the observations better. This type of assessment has its uses, particularly in the actual development of theory and observation, and is the basis of much of the discussion of the estimates of the individual equations in chapter 4. It does not however constitute a sufficient test of the model itself; because of the interrelationship between the sample data and the model this can be provided only by an examination of data outside the sample period, or prediction. There are other forms of nonpredictive assessment, such as the testing for acceptable dynamic properties in simulation, but being more concerned with internal rather than external properties these are usually considered secondary to the more obviously external tests.

'Prediction' is used to refer to statements about variables whose values have not been taken into account in determining the predicting function, and thus includes the case where the values to be predicted have occurred, and are even known though strictly ignored in the determination of the predicting function (as here), as well as the case of true forecasting when the values to be predicted are yet to occur. Predictions from stochastic models must also be stochastic, and their random elements will arise through disturbances both in the sample period (making the predicted expected value differ from the true expected value), and in the prediction period (making the actual value differ from the expected value). The statements called predictions must then be in the form of probability distributions, though for simplicity (at the cost of some information) these may be condensed into one attribute of the distribution, most frequently (as here) its expected value - the point prediction.

Prediction may be unconditional or conditional, conditional that is on the values of other (exogenous) variables. For practical purposes the greater the number of conditions attached to a prediction the less use it is; in the limit the prediction degenerates into a tautology. For assessing a model however, conditional predictions are of greater use than unconditional since certain sources of error outside the scope of the model are excluded, though the resulting assessment must be relative to the professed scope of the model - a natural constraint. Indeed the absolute assessment of a model is not meaningful; at best it can only reflect the assessment of the model relative to the assessor's implicit a priori model. The overall assessment of one model relative to another also means little in general, for it involves both the accuracy (A) and the unconditionality (U) of prediction - the latter being measured in some appropriate information units; it is only meaningful in the special cases where A (or U) dominates in either model, for in other cases it must depend on the assessor's utility function connecting A and U. This concerns the assessment of the accuracy and acceptability of the model as a true abstraction of reality; assessment for practical purposes may be made by extending one model until the same degree of U is reached for each.

5-2 Theory of solution

The assessment of the model as a whole then depends on the solution of the model as a whole, and as with the problem of estimating the model that of solving it is complex and may be approached in a number of ways. The problem of solution however differs from that of estimation in that whatever method is used the solution, providing that there is one, is the same (assuming that 'solution' means all solutions, to cover the unlikely case for an economic system of multiple solutions). In general one or more nonlinear equations cannot be solved explicitly; solution must thus be iterative, and it is for this reason that there are a number of possible methods. The solution of nonlinear equation systems is an important adjunct to analytical economics, and is a field which has perhaps received insufficient attention, despite the work of Holt [1965] on the Brookings model, and more generally of Holt et al [1967]; the problems posed have more frequently been avoided by imposing the constraint of linearity or by not solving the models at all. For this reason this section discusses some aspects of the theory of solution of general systems that are relevant to this study.

The various methods available are of varying degrees of power, a powerful method being interpreted as one which reaches a solution after a small number of iterations; as the more powerful methods do more at each iteration, they naturally tend to be more complex at each stage. Experiments were made with methods at each end of this spectrum, and it was found that for the type of nonlinearities and typical range of initial trial solutions encountered the more powerful (Newton Raphson) was significantly more successful in reaching a solution than the less (Generalised Newton) despite its lower robustness in general; it is thus used. This emphasises the fact that each model is likely to have its own most appropriate method of solution.

NR method The discussion of the Newton Raphson (NR) method here is not intended to be a complete analysis of its mathematical properties, but an outline of these sufficient for the understanding of the basic rationale and for the derivation of an efficient computation procedure. Computation was performed entirely by electronic computer for which a program (A NRSS) was specially written; a copy of its published specification is given in appendix C.

The NR method for solving a system of equations is best developed by first investigating the method for the solution of one equation

$$f(x) = 0$$

and then showing how the results may be generalised. The method starts from an initial trial solution $x^{(0)}$, and expands the equation in a Taylor's series about this, obtaining the next approximation as the value of this series truncated after the first derivative term. Thus for the k'th approximation

$$f(x) = 0 = f(x^{(k)}) + (x - x^{(k)})f'(x^{(k)}) + \dots$$

and $$x^{(k+1)} = x^{(k)} - f(x^{(k)})/f'(x^{(k)}).$$

This may be interpreted geometrically as the approximation of the curve by its tangent at the trial solution point, the next solution being given by the intersection of this tangent with the x-axis. For this method to be valid it must be shown that the solution exists, is (under appropriate restrictions) unique, and that successive stages of this process converge to this solution. This is proved by a theorem of Ostrowski's, which in a form generalised by Kantorovich is given in Saaty and Bram [1964]. This theorem places conditions on the initial value $x^{(0)}$, and ensures the existence of a solution by choosing an $x^{(0)}$ which will give the desired result, showing that max$[x^{(0)} - x]$ is nonzero providing that the function $f(x)$ is differentiable once in the relevant range. The restrictions placed on $x^{(0)}$ are not of direct use in suggesting an initial trial solution, but provide some information on the rapidity of convergence, which may be derived from the relation

$$\mathrm{mod}[x^{(k)} - x] \leqq \beta(2\alpha)^{2^k - 1}/2^{k-1}$$

where α and β are constants such that

$$0 < \alpha \leqq \tfrac{1}{2}, \qquad \beta \geqq \mathrm{mod}[f(x^{(0)}/f'(x^{(0)})].$$

In this connection it is relevant to note that convergence will still be obtained if a more simple method is adopted, that is if $f'(x^{(k)})$ is not computed at each stage but $f'(x^{(0)})$ is used in its place, so that

$$x^{(k+1)} = x^{(k)} - f(x^{(k)})/f'(x^{(0)}),$$

when the rapidity of convergence is given by

$$\mathrm{mod}[x^{(k)} - x] \leqq (1 - (1 - 2\alpha)^{\frac{1}{2}})^{k-1}\mathrm{mod}[x^{(1)} - x].$$

We now turn to the system of n equations

$$\underline{f}(\underline{x}) = \underline{0}$$

where $\underline{x}$ is the vector $[x_i]$, $\underline{f}(\underline{x})$ is the vector valued function $[f_i(\underline{x})]$, and $\underline{0}$ is the zero vector $[0_i]$, all for $i = 1, \ldots n$.

Again each equation is expanded in a Taylor's series about some initial trial vector $\underline{x}^{(0)}$, and the next approximation is the values of these series truncated after the first derivative term. Thus for the k'th approximation[1]

$$f_i(\underline{x}) = 0 = f_i(\underline{x}^{(k)}) + S_j((x_j - x_j^{(k)})df_i/dx_j^{(k)}) + \ldots,$$

and $$\underline{x}^{(k+1)} = \underline{x}^{(k)} - (\underline{J}^{(k)})^{-1}\underline{f}(\underline{x}^{(k)}),$$

where $$\underline{J}^{(k)} = [df_i/dx_j^{(k)}], \qquad \text{all } i, j = 1, \ldots n.$$

This may be interpreted in multidimensional space as the approximation of the hypersurface representing each nonlinear equation by the hyperplane that is tangential to it at the initial trial solution point, the next approximation being given by the vector corresponding to the intersection of these hyperplanes.

The proof of the existence, uniqueness, and convergence of the solution is best obtained by showing that the problem is analagous to that solved by Kantorovich for one equation and then using an abstract space generalisation of Kantorovich's proof; this is shown by Saaty and Bram [1964]. To prove that the iterations converge to the solution value in either case it is necessary to show that the absolute values of the adjustments at each stage, that is mod$[(f'(x^{(k)}))^{-1}f(x^{(k)})]$ for one equation and mod$[(\underline{J}^{(k)})^{-1}\underline{f}(\underline{x}^{(k)})]$ for many (the latter being a generalisation of the former), remain within convenient bounds at all points between the initial and final

1. The partial derivative is written as the simple derivative, 'd', as the latter is not used here; 'df/dx' is thus the partial derivative of f with respect to x.

solutions, and that the sequence of bounds tends to zero as k tends to infinity. This is equivalent to keeping a check on the 'distance' between successive solutions, as this is the adjustment at each stage. In the general case however there is no unique way of measuring this for the concept of absolute value used in the single equation case is not meaningful when applied to the vector difference $\underline{x}^{(k+1)} - \underline{x}^{(k)}$. To show that the multiequation case is a true generalisation of the single equation case it is necessary to choose some measure of this 'distance' of which the absolute value is a special case; clearly if this is to be interpreted as a 'distance' it must satisfy the general requirements of a norm. Saaty and Bram show that the maximum absolute value is an appropriate norm, and thus by replacing $\mathrm{mod}[f'(x^{(k)})^{-1}f(x^{(k)})]$ in the single equation case by the absolute value of the (absolutely) largest component of the vector $(\underline{J}^{(k)})^{-1}\underline{f}(\underline{x}^{(k)})$ in the multiequation case, the generalised version of Kantorovich's proof may be shown to apply to the general NR method.

This provides an introduction to the rationale of the basic method. Among the number of modifications to the NR method which have been suggested the most relevant here is that which introduces some damping to the adjustment at each stage, thus decreasing the probability that the next solution may overshoot in the sense of increasing the sum of squared errors. This replaces the algorithm

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} - (\underline{J}^{(k)})^{-1}\underline{f}(\underline{x}^{(k)})$$

by $$\underline{x}^{(k+1)} = \underline{x}^{(k)} - \gamma.(\underline{J}^{(k)})^{-1}\underline{f}(\underline{x}^{(k)})$$

where γ is a constant such that $0 < \gamma < 1$. By expressing the sum of squared errors as a function of γ and differentiating this with respect to γ, Holt et al [1967] show that at $\gamma = 0$

$$du/d\gamma = -2[\underline{f}(\underline{x}^{(k)})]'[\underline{f}(\underline{x}^{(k)})]$$

where u is the sum of squared errors, and thus as the value of this derivative is necessarily negative the sum of squared errors decreases as γ increases from zero. Since with $\gamma = 1$ this modification reduces to the basic NR method for which it was assumed that the sum of squared errors would increase, there must be some value of γ between zero and unity for which convergence, in the sense of decreasing sums of squared errors, is assured. Thus this modification relaxes to some extent the restrictions placed on $\underline{x}^{(0)}$, and may be important practically - though damping will generally entail slower convergence; the modification is incorporated where necessary in the method used (see appendix C).

The NR method and its variants use information from all the equations for each modification of a trial solution, and thus involve possibly lengthy matrix inversions. This is its main strength and weakness, and distinguishes it from most other methods, which attempt to avoid matrix inversion by solving each equation in turn. These methods may be particularly suitable for solving economic models, which are typified by having only mild nonlinearities and being reasonably sparse - for example the 150 equation Brookings model rarely has more than **six** (endogenous) variables in an equation. For this reason one such method is briefly examined.

GN method The generalised Newton's method (GN) proposed by Greenspan [1965] is essentially a degenerate form of the NR method, where the resulting error in one equation at each stage in the process is multiplied by the reciprocal of a single derivative to give e adjustment to the next stage, instead of multiplying the vector of errors by the inverse of the whole Jacobian matrix. This requires that the equations be arranged in a particular order so that x_i appears, preferably with a high 'weight', in the i'th equation, that is that df_i/dx_i is nonzero, and preferably relatively large in absolute value, for all i (i = 1, . . . n). The iterative procedure is then given by adjusting each stage in the manner described above, using the latest values of all x_i at each stage. This produces the algorithm

$$x_i^{(k+1)} = x_i^{(k)} - \gamma f^{(i)}/(df^{(i)}/dx_i),$$

where $$f^{(i)} = f_i(x_1^{(k+1)},\ldots x_{i-1}^{(k:1)},\ x_i^{(k)},\ldots x_n^{(k)}),\quad i = 1,\ldots n,$$

γ again being a damping factor. It is a possible criticism of this method that it requires all df_i/dx_i to be nonzero, and preferably large, though in economic systems there is frequently a normalised dependent variable which may be taken as the main variable in each equation; the question of assessing importance is however awkward, for in many cases the weight of a variable is arbitrary and depends on the units used. A further difficulty is that the method appears to be very sensitive to the value of the damping factor γ; Greenspan gives many examples of failures to converge arising from a slightly wrong choice of this factor. The method does however have the advantage that it requires no matrix inversion and may thus be particularly useful for large sparse nonlinear systems; it may also be preferable to an explicit method for solving large linear systems. For these reasons the GN method was investigated here, but found to be inferior to the NR, and so abandoned.

5-3 Solutions of the industry models

Before discussing the actual solutions of the industry models this section makes some comments on the scope and method of the particular tests in this study; these comments apply equally to the next section.

Scope and method It is important to note that, for the reasons mentioned above, solutions are only given for four time periods, and thus strictly these only provide a tested methodology for the assessment of the models and the nature of their interaction, and indicate some tentative general results; these solutions then provide a necessary rather than a sufficient test for the accuracy of the models. This is equivalent to saying that the statistical significance of results based on four observations is difficult to assess; this is however a matter of degree, and in an inexact science such as economics apparently strong and consistent results from four observations may well be (and frequently are taken to be) enough to show some interesting tendencies. Thus must be the nature of any conclusions drawn.

The question of assessing the statistical significance of a few results is made considerably easier if the results, or predictions, are in the form of frequency distributions rather than being point predictions, for then sample variances are given and need not be inferred from the variance of the sample means. Only point predictions however are presented here, without any estimates of the standard errors of forecast; this is because the latter depend in a complex way (for nonlinear models) on the variances and covariances of the estimates of the parameters of the model. Further, not all the latter are known, for the estimation method used does not take account of the specification of any equations in the model other than the one being estimated, and so the covariances between estimates of parameters in different structural equations are not directly obtainable. Goldberger et al [1961] show how approximations to these may be obtained, but this method is not followed due to the greater complexity involved in a nonlinear system.

The solutions sought are those of the model presented in appendix D, prediction being used to assess the explanatory power and accuracy of the basic model not to assess its forecasting power per se, when any modification which genuinely improves this would be acceptable. Accordingly it is assumed that the disturbances in all the structural equations take their expected values of zero. This has two consequences; firstly it does not allow predictions to be improved by taking account of the sample period serial correlation of disturbances, and secondly, although the disturbance in each equation individually assumes its expected value, the predicted values of variables generated by simultaneous solution may not be the true expected values. The first of these may be desirable if the hypothesised model is free from serial correlation, but is inappropriate if the specification of the model includes that of its serial correlation. This model attempts (though not completely successfully) to explain the relevant phenomena without the (explicit) use of such autoregressive structures. Assessment here is thus the more rigorous assessment of the more fundamental structure. The second of these is undesirable, and is accepted only because of convenience and because the distortion so caused may be expected to be small. This distortion arises through the nonlinearity of the system; the expected value of a linear function of a set of random variables (the stochastic solutions of individual equations) is this function of their expected values, but this is not necessarily the case for a nonlinear function.

The industry solutions are concerned with the acceptability of the industry models in isolation, providing both a means for choosing between the two models retained at this stage and an 'absolute' assessment of the model so adopted. It is recalled that the two models are firstly that with the deterministic relation for profits (MA), being equations 1 - 7, 8a, 9, 10 of the outline industry model, and secondly that with the stochastic relation for profits (MB), being equations 1 - 7, 8b, 9, 10. Each model (MA and MB) for each industry is provided with the true values of all the link variables it uses, as well as all the predetermined variables, and solved simultaneously by the NR method. The errors in the predictions of output and profits (as being the most interesting of the ten variables predicted by the ten equations) so generated by each model for each industry for the four time periods are presented in appendix E; also presented are the corresponding figures for the aggregate and root-mean-square (RMS) errors, which are derived from the individual industry figures. These provide two means of summarising the industry figures, of which the RMS figure is the more interesting for the assessment of a set of industry models which are meant to be individually accurate; this then is taken as the most important summary measure of the accuracy of prediction.

It was shown above that absolute assessment has no meaning, and that assessment may only be relative though may be considered to be quasiabsolute if it is relative to some simple, meaningful, and generally acceptable alternative. The alternative here is the frequently used naive model, and so the corresponding errors generated by the three most general types of naive model are also presented in the appendix for comparison. These naive models are: firstly the simple naive model (NS) where the estimate x* of the current value of a variable x is its last value,

$$x^* = x_{-1};$$

secondly the linear naive model (NL) where the estimate of the variable is its last value plus its last absolute growth,

$$x^* = 2x_{-1} - x_{-2};$$

and thirdly the geometric naive model (NG) where the estimate of the variable is its last value times its last proportional growth,

$$x^* = x_{-1}^2/x_{-2}.$$

These three naive models might be appropriate where the variable is expected to fluctuate about some steady value, some linear trend, and some proportional growth path respectively. Other naive models could be proposed, but anything more naive is unlikely to be accepted as meaningful, and anything less as simple. The use of such naive models depends on the choice of a suitable variable, for the result of testing an equation connecting several variables against a naive model will depend on the normalisation of the equation; output and profits would appear to be acceptable variables here in view of the aims of the model. The superior performance of a basic model to a naive model may be considered a rather weak property, for if prediction is considered as the acid test this is almost subsumed in the term 'model'. On the other hand this may indicate some real achievement, for underlying processes have presumably been laid bare as well as a superior predictor being provided for the real world; indeed various areas of economic analysis, notably input-output analysis, have at least in their infancy been surpassed in predictive power by naive models. This apparent dichotomy may arise through the differing amounts of information used in different models; a 'reasonable' test must compare with the naive model a model which uses only 'reasonably available' information - so for example it is of little meaning to assess the prediction of total income if consumption and investment are treated as given. There is however no real objective test of what is 'reasonably available', so it is important to take account of the amount of information used. The tests here may thus be open to objection since they use true values of the link variables. This may be acceptable methodologically since the tests are tests of the industry models in isolation and this is the most valuable way of comparing MA with MB - since extraneous error is removed; in practice the overall results hold even when the true values of the link variables are replaced by naive estimates of their values, as is shown below. The results of these tests may thus be interpreted as at least an indication of the acceptability of the industry models.

Results The details of the results of solving the industry models are given in the appendix and discussed here. These are summarised in table 5-1; this gives the accuracy of RMS profit predictions in each period for MA and MB using the true values of the link variables, for the best of the three naive models, and for MA using lagged values of the link variables.

Table 5-1: Summary industry results

	Accuracy of RMS profit predictions (percent)			
Period	MA (true)	MB (true)	Best naive	MA (lagged)
1	15·2	55·5	17·3	13·3
2	19·7	42·5	36·5	17·6
3	14·6	32·3	15·9	22·6
4	22·5	50·3	19·7	23·3

The first apparent result is that the prediction of disaggregated quarterly profits is extremely difficult; this is reflected in the (subjectively) high errors produced by all models, which are associated with the high variances (relative to the mean) of nearly all profits series, as is typical of a variable which is a (small) residual. Further, both models behave reasonably, in that they do not produce solutions which are obviously absurd; this does not say much, but is a necessary condition for further tests.

The second result is the important one that MA is substantially superior to MB in terms of profit prediction. MA is more accurate fairly consistently for the industries separately than MB, and this is summarised in the consistently larger RMS error for MB for all four periods, this being of the order of two or three times that for MA. In aggregate terms as well MA is substantially better than MB. Given the aims of the model it is reasonable to be primarily concerned with profits, though output (and the other variables not listed) are also relevant; at the moment however no mention need be made of these, for as current profits appear only once in the industry models there is no immediate feedback effect and predicted values of all other variables are the same for MA and MB. This does not imply that profits have no effect, but only that the effect is through the overall interaction of industries or delayed beyond the current period (as indeed is to be expected). We conclude then that MA is clearly preferred to MB, and is adopted; all future mention of 'the model' refers to MA.

The third general result at this stage is that the industry models are, given the information they use, good predictors of disaggregated quarterly profits, and of industry behaviour in general. This result is clear in all four periods except the last, where it is difficult to draw any conclusions. If any period is to be dubious it may be expected to be the fourth, for it is a quarter of particularly violent change, as is apparent from a casual inspection of the data series (for example unemployment in industry v takes the values 6, 7, 8, 71 over the four quarters - there were no unusual industrial stoppages). Figures for profits produced by the model are better than those produced by any naive model for nearly all industries for the first three periods, and for half of the industries in the fourth. This is a reasonably stringent test for it compares one real model with the best of three naive models, and although on the whole NS is better than either of the other naive models (as might be expected for a series fluctuating widely about a fairly steady level), for certain industries in certain periods NL or NG is best, and the test requires that the real model performs more satisfactorily than the best of these three in each situation individually. If only one naive model is adopted for all situations the superiority of the real model will appear more marked. Overall in RMS terms the real model is better than the best of the naive models in the first three periods (substantially so in the second), and slightly worse in the fourth. In aggregate terms however the best of the naive models is better than the real model for three of the four periods, but this is natural as the aggregate of any series may be expected to be more stable (and thus amenable to prediction from a simple time series) than its components; this may be particularly true for profits where a gain by one industry may be in part the loss of others. Thus naive predictions may be expected to be superior as (for NS and NL - the two most relevant) the aggregate of naive predictions is equivalent to a naive prediction of the (relatively stable) aggregate, for these models are linear and thus may be validly added. Output figures are perhaps of less interest at this stage because of their more direct connection with the (true) link variables; they are however better overall in terms of RMS and aggregate figures for the real model than any naive model.

The industry models may thus be considered to be satisfactory for incorporation into an overall model which generates the values of the link variables; this answers the main question posed by these tests. It does not however answer the separate and interesting question of whether they are in an absolute sense useful in themselves, or whether their apparent superiority to naive models in predictive ability is merely due to the information (in the form of true values of link variables) that they use. From the discussion above it is clear that this question may be answered by extending the industry models so that only some 'standard' amount of information is used, this being taken as only lagged variables and true (policy) exogenous variables (here - with little importance - government expenditure on goods, government capital expenditure, and bank rate). The simplest way of doing this, that is the way which changes the basic models least, is to postulate simple naive models (NS) for the values of all link variables, or in effect replace all true values of link variables by their lagged values. Such solutions are produced as a byproduct of the solutions of the model as a whole (discussed in the next section), the overall results being indicated by the solutions produced by the first iteration of the entire model, as given in the second part of the appendix and summarised in table 5-1. It is apparent that replacing the true values of link variables by their lagged values has various effects, but does not significantly affect the basic result that the predictions of the real model are superior to those of any of the naive models. In terms of the basic summary statistic RMS error of profit prediction for example, predictions are actually better with lagged values rather than true values of the link variables for the first two periods, worse for the third, and approximately the same for the fourth. The indication is then that predictions are of approximately the same accuracy when true values of the link variables are replaced by lagged values, and thus that the models are useful predictors in a real sense.

5-4 Solutions of the model as a whole

It should be recalled that the general introductory remarks of the previous section also apply to this section. The solutions discussed here are concerned with the interaction of the industry models and the quantitative nature of the tatonnement process - particularly whether this achieves acceptable convergence of the system.

Scope and method The procedure is an iterative one in two stages, and consists of postulating some initial values for the link variables, solving the 10 ten-equation industry models seriatim by the NR simultaneous (iterative) method, and using the values of variables so generated to form new values of the link variables, either directly or through parts of the economy model, and repeating the process until convergence of the whole system is achieved (or appears impossible). Each iteration or stage in the process generates a complete set of solutions to the models at that stage, which are summarised in appendix E. The accuracy of these (intermediate) solutions for the industry models separately is indicated by both the RMS and aggregate errors of prediction for profits and output; also indicated is the accuracy of each prediction of the mean value of gross domestic product, that is the average of total product and total income, or $\frac{1}{2}(Y^{i*} + Y^{p*})$. Total income and total product are of course ideally identical, but data inaccuracies make this irrelevant unless arbitrary steps are taken to remove the residual accounting error. As this study is to some extent concerned with the consistency of the model it is useful to have some indication of how much inconsistency arising through inaccurate data is acceptable, and so the residual error has in effect been retained, that is the figure used for both total income and total product are those that are considered individually to be the most accurate. A measure of the consistency of each solution is also given; this is the difference between predicted total income and product expressed as a fraction of the actual difference, or $(Y^{p*} - Y^{i*})/(Y^{p} - Y^{i})$. Finally an indication of the degree of convergence of the system at each stage is given, this being the RMS percentage (thus removing any scale effects) difference between individual values of the link variables after each iteration; this can only be zero if no link variable changes from one iteration to the next, when the system is temporarily in equilibrium.

Essentially this procedure is very simple; it is a generalised tatonnement process whereby all agents in the economy make their decisions given the general link data at each iteration, there being no activity until an equilibrium set of link values has been generated. The initial values adopted (Walras' cries hazard) may as well be sensible rather than purely random, yet clearly should not be based on any information not (ideally) available to the various agents, such as the true values. The obvious choice on both criteria is the last period's values, and so (as indicated above) these are used.

Though theoretically simple the procedure is more complex computationally, for it concerns a large nonlinear system. It may be recalled that there are 10 industries each with 10 equations (one of which is discontinuous, involving the solution of two subequations and the adoption of the maximum solution, all within a simultaneous nonlinear environment), and approximately (the exact number depending on the number of definitional relationships counted) 28 economy equations (14 stochastic and 14 deterministic), making a total of about 128 equations. Each industry uses (in effect) 8 link variables, making about 80 link variables at each stage; in practice many of these are duplicated, that is are the same for more than one industry, but to allow these to assume greater importance these 80 are treated as being independent in measuring convergence - thus the percentage differences for those appearing more than once are weighted accordingly. In this procedure then the ten industry models are each solved by the NR method at each stage, and the economy equations are solved (given the values generated by the industry models) seriatim, as they may, with one exception, be broken down into a recursive ordering which avoids the need for simultaneous solution. The exception is the block of equations determining consumption and consumer prices, for consumption depends linearly on relative prices and thus nonlinearly on aggregate price, while aggregate price is the sum of the individual prices weighted by the individual consumptions. This simultaneity is of relatively minor importance since it arises only through the need to weight the various components of an aggregate, and thus the problem is artificially avoided by using lagged instead of current values of consumption as weights to calculate the aggregate price, and making an approximate check by recalculating this from the current values of consumption thus generated. The two figures are always very close, the maximum difference being of the order of one half of one percent, and thus the approximation is accepted in practice.

Computationally then the problem is large, and no great numerical accuracy can be expected; complete convergence in practice is then unlikely even for an ideal model of this size.

It is not easy to quantify this, but it would appear that a degree of convergence as defined above of around five percent should be taken as being very satisfactory (given the size of the system and the machine used), and that an ideally convergent system would be expected to converge to somewhere around this level and then undergo mild damped oscillations around this level.[2] We thus define convergence as occurring in practice at the iteration which corresponds to the first minimum of the degree of convergence defined above that is at a degree of less than five percent provided that successive iterations appear to converge to some degree of less than five percent; if the convergence path is monotonically decreasing convergence is defined as occurring at the first iteration that produced convergence of a degree of less than five percent.

Results The full results of fifty iterations of this process are given in the appendix, and are discussed here; they are summarised in tables 5-2, 5-3, and 5-4.

The immediately apparent result is that the process converges very rapidly, exceeding all expectations. More specifically, convergence as defined in the first sense above occurs after about five iterations, it is at a level of around one percent, and further iterations suggest that the convergence path may become asymptotic to a level just below a half of one percent, even from initial values which are inaccurate enough to produce initial factors of up to two hundred percent. These figures are summarised for the four

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2. This is the view of a systems designer for the machine (KDF 9) and cannot readily be objectively supported; it serves however as an approximate indication of the accuracy to be expected.

individual periods in table 5-2, from which it may be inferred that this simple tatonnement process is very effective.

Table 5-2: Summary results on convergence

Period	Convergence iteration	Convergence (percent) Achieved	Asymptotic	Initial
1	7	1·1	0·4	16·3
2	5	0·7	0·4	80·6
3	6	0·9	0·3	74·4
4	3	1·5	0·5	204·8

A second apparent result is that the convergence solution produces consistently better figures for total profits than the initial solution, but is not necessarily better in general; these profit figures show no consistent bias, but figures for output, or total product, show an apparently consistent downward bias. In general the initial solutions are too high relative to the true values, and subsequent iterations produce decreasing values, passing through the region where inaccuracy is zero, this region of peak accuracy usually lying between the initial and convergent iterations. This downward drift as iterations proceed occurs in the accurate profit figures as well as the downward biased product figures. This is summarised in table 5-3, which gives for each period the convergence (Cve) and initial (Inl) accuracy together with the iteration (optimal) closest to absolute accuracy for both total product and total profits (though this is merely the first if the initial accuracy is negative); for reference the convergence iteration is also given.

Table 5-3: Summary results on accuracy (percent)

		Product			Profits		
Period	Cve iteration	Accuracy Cve	Accuracy Inl	Optimal iteration	Accuracy Cve	Accuracy Inl	Optimal iteration
1	7	-2·0	5·5	6	-3·9	11·3	5
2	5	-8·2	-4·8	(1)	0·5	7·3	5
3	6	-4·0	1·5	3	5·2	16·6	9
4	3	-2·5	-0·9	(1)	-0·2	0·9	3

It thus appears that the converged tatonnement solution indicates substantially better (and unbiased - indeed absolutely good) predictions than the initial solution for total profits,but slightly worse (and downward biased) predictions for total product. The conclusion concerning total product appears valid, but that concerning total profits must be severely qualified by the higher RMS error for profits, indicating that this accurate total is made up of offsetting inaccurate components. The individual figures show this, and also show a consistent pattern in each period: figures for (the large) industry n are always too high, and those for industries e and v are always too low; figures for other industries are predominantly too low, counterbalancing the large excess effect of industry n. We may then conclude that the equilibrium convergence solutions tend to be consistently lower than the true figures for total product by a factor of the order of five percent, and similarly lower than the true figures of profits in most industries, though a higher figure in nonmanufacturing makes the total very accurate.

It is relevant here to note that as iterations are taken beyond the convergent iteration all solutions become steadily lower. This however means little, for if the above definition of convergence is accepted then these subsequent solutions have little relevance: they are merely the result of cumulatively subtracting at each iteration the discrepancy associated with (approximately) the asymptotic degree of convergence. Clearly if a small discrepancy in the definition of convergence is accepted, then each iteration beyond the convergence iteration must differ from the one preceeding it. It is not surprising that the changes are all in the same direction, for theoretical work on the stability of simple (price) tatonnement processes (as summarised by Negishi [1962]) shows that these converge monotonically; it is plausible to suppose that the more general process adopted here is similar. This would produce the apparent cumulatively increasing 'inaccuracy' of later iterations. Iterations after the convergent iteration are only performed to ensure that the process does not (within a reasonable number of iterations) later diverge.

A third result concerns consistency as defined above, where with one exception it is apparent that all solutions have a degree of consistency of the same order as the true figures; this is true for early iterations (from the initial to the convergent) and also of most later iterations. The exception is the second period, where the consistency of later iterations appears very poor; this however is because the true figures for this period are highly consistent (an occurrence which may well have come about by chance), so the measure reported is the result of dividing the actual inconsistency by a factor which is near zero. This then merely shows the inadequacy of using a random variable whose expected value is near zero as a deflator; it is used partly because the particular figure for the period may be important, which precludes the use of the mean or RMS figure over many periods, and partly because the measure is not intended as anything more than an approximate indicator. It is apparent that the convergence solutions are of the same order of consistency as both the initial solutions and the true figures (whose consistency is defined as unity). Similarly to the indicators of accuracy, the indicator of consistency for all periods is positive for the initial iteration and then decreases fairly steadily through zero as further iterations are made. Thus it may be concluded that the model is likely to produce, in the sense used here, consistent predictions - though this is not unexpected for data so close to the sample period. These results are summarised in table 5-4, which gives for each period the convergence and initial consistency factors (as defined above), together with the iteration (optimal) closest to absolute consistency; for reference the convergence iteration is also given.

Table 5-4: Summary results on consistency

Period	Convergence iteration	Consistency factor convergence	Consistency factor initial	Optimal iteration
1	7	2·0	3·6	14
2	5	-6·2	4·2	3
3	6	-0·4	1·7	5
4	3	-0·4	0·2	2

This explains why there is no investigation of the path to consistency. The lack of this does not however imply that there is no overall constraint to the solved model, but that this constraint is stochastic rather than exact, and is immediately (approximately) satisfied by the solutions obtained. If the constraint were not immediately satisfied the mechanism for its application would be as follows: a convergence solution is obtained as above and tested for consistency; if this is not consistent a new hypothesis is made about the value of the disturbance term in the profit relations in each industry, and the process repeated until acceptable consistency is obtained. The fact that the profit relations have been reduced to fixed proportions does not preclude an error term under conditions of imperfect data, for the whole question of consistency would not arise if the data were made to fit the desired identities and fixed proportions. These changes would be equivalent to the addition of a constant to the profit relations, these constants being distributed according to the relative levels of profits in the industries, where the total distributed is the overall inconsistency gap, that is total product minus total income (or some suitable function of this). Thus at stage k profits $z^{(k)}$ in each industry would be replaced in stage k + 1 by

$$z^{(k+1)} = z^{(k)} + ({Y^p}^{(k)} - {Y^i}^{(k)})\ z^{(k)}/Z^{(k)}.$$

This would stress the role of profits in achieving consistent solutions. As however the inconsistency gap is a random variable and there is no indication that the inconsistency in convergence solutions arises through anything more than the inconsistency of the data as illustrated by the true figures, such an exercise would be meaningless and is not attempted.

CHAPTER 6 CONCLUSIONS

This chapter draws together the results which have become apparent in the last chapter, and examines some implications of these; some possible extensions and final remarks are then presented. The nature of this study is that it does not seek to answer a specific question, but to develop and test a model which may then be used to answer specific questions; for this reason the conclusions presented here do not necessarily indicate the total extent of the study.

6-1 Implications

The main results of the solutions of the models may be summarised under three headings. Firstly, the industry models are realistic, and of the two forms proposed that incorporating the deterministic profits relation is the superior; this produces absolutely good predictions of quarterly disaggregated profits, both in an artificial sense when it uses illegitimate information, and in a real sense when it uses only legitimate information. Secondly, allowing for the practical difficulties of computation, the simple tatonnement process proposed solves the complex equilibriating equations between industries both efficiently and consistently. Thirdly, the convergence solutions produced by the tatonnement process tend to be slightly but consistently lower than the true values for both output and profits, except in non-manufacturing.

Accuracy An implication of the first of these results is that the model presented may form the basis of practical policy or forecasting models which seek to determine future values of this difficult and important variable, quarterly disaggregated profits. As discussed in chapter 1, it may be argued that profits provide the mainspring of change in a dynamic and efficient economy, whoever received them: that is in a socialist and a capitalist economy. As one of the prime facets of economic efficiency is efficiency in allocating resources between different areas of production (industries) with reasonable rapidity (say within a quarter), a practical search for economic efficiency may be helped by having some acceptable predictions of quarterly disaggregated profits. There is no directly comparable work against which the accuracy of the models presented here may be judged, but in view of the absence of such work and the performance of the models relative to naive models, it is not unreasonable to suggest that these may provide at least a basis for more practical models. Modifications would however be necessary, particularly to take account of the delay in obtaining recent values of some predetermined variables; these are outside the scope of this study. A further implication of this result is that the quasibehavioural approach to the determination of profits as proposed by Evans [1968] and discussed above appears significantly inferior to the simpler identity approach in terms of prediction from some standard amount of information.

Convergence The second conclusion may be of relevance to the question of stability in a competitive economy or of simulating the competitive equilibrium, and thus the social optimum, in a socialist economy, and provides conclusions which cannot be drawn from deductive logic, or pure theory, alone. The solutions show that the actual system, as synthesised in the system of equations forming the model, can and does reach equilibrium very rapidly by acting according to some very plausible and simple algorithm; they do not then merely show that the system has a mathematical solution, but also show how the system itself can (rapidly) 'grope' its way to this solution. The advantages over a purely theoretical approach are threefold; firstly the model represents the actual economy, which differs substantially from a competitive economy, and so the results though more particular are more realistic than those derived from a theoretical study of a competitive economy. Secondly, the actual path towards equilibrium is shown and the number of iterations required is produced; quantification is as relevant here as in any other field of economic science. Thirdly, stability is shown from what may be interpreted as the real initial positions, which theoretical results clearly cannot do. If a linear system is shown to be stable locally then clearly it is stable globally, for the two properties are equivalent. Linear systems however can never hope to be truly representative of economic processes, and the local stability of a general system does not imply its global stability; theoretical approaches cannot usually show the quantitative limits of the range in which local stability occurs, and thus produce no direct results on the nature of stability from actual starting points. This study then produces quantitative results on the nature of the stability of an actual economic system when started from realistic initial values, though it is inevitably particular, and rather limited in its practical scope. It suggests sufficiently interesting quantitative results to infer that this may be a promising line of investigation.

This then is perhaps the most important prescriptive implication of the study, for the stability of the tatonnement process in a noncompetitive economy suggests that a decentralised planning procedure based on such a process may well work most efficiently in such an economy. It will be recalled from chapter 1 that all that is required conceptually for such a procedure to be successful is that the process converge - and on a more practical level, converge reasonably quickly. This result then indicates that the tatonnement process converges (to within practically acceptable limits) in approximately five iterations, and thus that it might provide a suitable basis for some actual decentralised planning procedure in a socialist economy.

It is suggested that the more general tatonnement process postulated here, where other variables besides price have a parametric function (that is are a resultant of the behaviour of all individuals though are regarded by each as given data to which he must adapt), helps in achieving the rapid convergence shown here. It would be interesting to investigate this, but the essentially interwoven nature of the industry models means that this cannot be done without rebuilding the whole structure.

It is perhaps unfortunate that the size of the computational problem posed and the resources available mean that the convergence solutions are not exact. Economics however is an inexact science and the acceptability of this type of inaccuracy is not absolute but depends on its degree; the error here is small relative to other types of error, such as the random disturbance component in every stochastic equation, and may thus be acceptable. Alternatively the process may be interpreted as a type of quasitatonnement, where the system gropes its way to an approximate solution, whereupon all individuals act.

Clearly the relevance of this result depends on the acceptance of the two assumptions that the industry is the individual agent and that the time period of a quarter is a short one - approximately equivalent to a Hicksian week. Like most other valuable assumptions in economics these are not perfectly satisfied, though they are perhaps of sufficient validity to be useful. The ten industries are taken to approximate to the individual agents or firms for practical convenience, and a time period of a quarter is adopted for similar reasons, being the shortest period for which data is generally available; these matters were discussed in more detail in chapter 2, and the assumptions must be considered as inevitable limitations to the applicability of the model.

Distortion   The third result has a possibly important interpretation, though it must be emphasised that on the basis of the information available this can only be tentative.

The overall model consists of a number of industry models connected by a skeletal economy model. All these are accurate in the sense that they produce reasonably accurate predictions with no clear consistent bias. This is demonstrated for the industry models by the first result above. It is also true for the economy model for this consists predominantly of identities and fixed proportions that are completely accurate, and also of behavioural equations which fit very well during the sample period (as shown in appendix D) and predict very accurately outside the sample period; these purely economy predictions are not presented as they are, as expected, very accurate, and are of less interest than the industry predictions.

The convergence solution arrived at by the tatonnement process is the competitive equilibrium position between the individual agencies, or industries, for this is the solution that would exist if each industry were to treat all variables external to the industry as given parameters which could not be influenced by the industry. Thus if there were perfect competition between industries (we ignore for the time being imperfect competition within industries) the actual solution of the system would be that arrived at by the tatonnement process, and with the basic assumptions of welfare economics, the social optimum. Now if the tatonnement solution is not the same as the true solution (as we are concerned with a stochastic system 'not the same as' means 'consistently in direction and magnitude different from') this must be due to one of two causes: either the models are inaccurate, or monopoly elements are acting so as to make the system arrive at a noncompetitive, and thus socially suboptimal, equilibrium position. As we have suggested that the former cause is not applicable we may tentatively infer that imperfection competition between industries exists, and that it acts so as to produce a social suboptimum.

This then is monopoly exploitation, though as it is apparent that both factors, labour and capital, exercise monopoly power to some degree and may thus disturb the equilibrium position from its social optimum, it is not clear whether the exploitation is due to labour or capital concentration. It is most important to note that this distortion or exploitation is such that it increases total product and total income above its optimal level, for the tatonnement solution is biased downwards; that is that society is being exploited by monopoly elements, but being exploited in the sense that its income is above the desired level. This result is not, as it might appear, absurd, for there is no fundamental reason why exploitation should decrease anything other than welfare. The concept of monopoly capitalism always decreasing output is relevant only in a narrow sense; it applies to monopoly within an industry and ignores the overall effects of a monopoly structure at the economy level, particularly in the factor rather than the final product markets.

A possible explanation of this phenomenon, which also makes it clear where the exploitation originates, is as follows.

We divide the population into three productive classes, which may be mixed: labourers, capitalists, and dependents. The third of these is for completeness only as it is by definition the null productive class; alternatively we may divide 'households' into the two main productive classes, and allocate each member of the dependents class to one or more of these. These two classes are similar but not equivalent to Marx's proletariat and bourgeoisie, for labourers are all those receiving an income from labour, whether this is a wage or a salary, explicit or implicit, and thus include a part of the bourgeois class; capitalists are those who receive an income from the ownership of any fixed resource, that is those whose income is independent of their personal (labour) input. These classes are not mutually exclusive for there will be a number of households whose sources of income are mixed, but as we may in general assume that these maximise their interests in each class independently this presents no problem; the two classes are however exhaustive, for income is defined as falling into one of these categories.

We assume that individual members of each class act so as to maximise their utilities, and make some elementary assumptions about these: firstly, that the utility associated with income is a positive increasing function of income, and secondly, that the utility associated with work (measured as a fraction of the total work possible in the period - in terms of time and effort) is, beyond some level which has been reached, a negative decreasing function of work. These are very weak requirements; we need no assumptions of cardinal measurement, decreasing marginal utilities, or absolute levels -

only that more income is desirable, and that less overwork is desirable. We do not assume that all work is undesirable, only that work beyond some level (which has already been reached), that is overwork, is undesirable. It is plausible that developed economies are in this overwork range, for few labourers would not work less if this did not entail a lower income - though a change in the labour-income relation due to accumulation of capital may well bring many economies into the desirable work range in the foreseeable future. The interest of the capitalist then is simply maximum income, for this is the same as maximum utility; the interest of the labourer however is to achieve a balance between income and work (or its obverse - leisure), and this entails making the utility of the last unit of income just balance the (negative) utility of the last unit of work from which this unit of income is derived - we assume labour income to be an increasing function of work. This is the familiar classical solution, which though too narrow to be general in that it assumes that work rather than overwork is undesirable, is valid within the range relevant here.

Monopoly power at the economy level may be exercised in two ways: it may either affect the total product, or it may affect the distribution of this between classes. The 'struggle between the classes' has always been primarily concerned with the division of the total product, and as the strength of monopoly elements on each side are of the same order of magnitude and are both actively concerned with the struggle, the net effect is negligible; that is the optimal competitive situation remains approximately if equivalent monopoly elements are introduced on both sides, it just becomes less determinate. The situation is very different however in the determination of the total product, for it is evident that in a capitalist economy this is determined almost entirely by the capitalist class; this indeed is the function of the firm. This does not deny the importance of effective demand, for production decisions must still be influenced by this; the main determinant of effective demand is however income generated, and any fluctuations caused by effective demand will be fluctuations about some new level. This then is where monopoly exploitation may arise, for (implicit) collusion may exist within the capitalist class to force the level of total product above its competitive equilibrium level (for example by making employment for less than the standard time almost impossible), whereupon with the same division of the product both profits and earnings increase, so the utility of capitalists increases while that of labourers decreases - since work is increased and by definition the competitive equilibrium or social optimum level of work was the maximum utility level.

This hypothesis is supported by the convergence solutions obtained; the tatonnement solution is one of lower product, profits, and earnings than the actual position, yet the factor distribution of income is approximately unaltered, except in nonmanufacturing. The exception of nonmanufacturing is tentative (it is but one industry out of ten, and the result may have occurred by chance), but if valid strengthens rather than weakens the hypothesis. This is perhaps the industry whose structure is most competitive, or atomistic, and so collusion cannot so readily extend to nonmanufacturing. The lower profit of nonmanufacturing would then be explicable in terms of the expected gain by manufacturing at the expense of nonmanufacturing when income is substituted for leisure.

This then is a possible implication of the third result, which, it is emphasised, must remain tentative; it is examined more because of its interest than its uncontrovertable nature, and as an example of the type of result which may emerge from this kind of analysis. It does of course depend on the fact that the convergence solutions of the quasitatonnement process postulated above are not necessarily the mathematical solutions of the entire system, and is thus in a sense an alternative to the numerical inaccuracy explanation of this phenomenon. More detailed results on the inefficient allocation of resources because of monopoly distortion, for example as between uses rather than overall (other than the tentative one concerning nonmanufacturing), cannot really be drawn from the rather limited amount of information generated in this study; in particular no inferences can be made about the effects of imperfect competition within industries, as each industry is treated as one agent. It is suggested then that monopoly capitalism at the economy level produces a suboptimal allocation of resources (the use of labour) and is thus inefficient, though inefficient in that it produces too much.

6-2 Possible extensions

There are many areas of this study in which improvement may be possible. These have on the whole been mentioned in context, but may be briefly summarised here in three groups.

Firstly, a large disaggregated quarterly model requires a large amount of data, and the collection of this must precede any extensions to the structure of the model. This is one perhaps unexciting but important field for development, particularly in connection with data for profits.

Secondly, there is almost always scope for the refinement of the structures of econometric models, and theoretical developments may well suggest various changes. Besides refinement, or improvement with the same number of variables, extension may well prove fruitful, and this may be either of the complexity, or number of equations, of the industry models, or of the disaggregation of the industries. Though this may be very valuable in certain areas, it should not be pursued for its own sake; the particular areas where extension may prove most valuable are the introduction of hours worked in the first case, and the disaggregation of the nonmanufacturing industry in the second.

Thirdly, there is a real need for further work with the solved model, both as regards a deeper investigation of the properties of the model by letting it run for several periods and possibly introducing random shocks, and also as regards obtaining checkable predictions and investigating the practical properties of the tatonnement process over a larger number of periods; this would allow more definite conclusions to be presented than the necessarily somewhat tentative ones here. This is perhaps the most important field for further study.

6-3 Concluding remarks

This study has proposed a development of quantitative economic analysis concerning the relationship of the individuals to the whole in an economic system; more specifically it has sought to explain one of the main motivating forces in a dynamic economy, short term disaggregated profits, and the process whereby the economy reaches a temporary equilibrium indicated by such forces. The study has been primarily concerned with presenting and testing a suitable methodology for this, and to this extent it has been essential to produce quantitative results; these however must primarily be considered as indications of the type and order of conclusion which may be forthcoming from this type of approach, rather than as accurate results in their own right. Within these limits the study may have proved successful: the industry models postulated have produced good predictions, the tatonnement process has produced rapid convergence to a consistent equilibrium and thus may be suitable for decentralised planning, and it has been possible to suggest the interesting result that the capitalist system is inefficient in that it produces too much.

APPENDIX A REFERENCES

Abbreviations of authors

CDAE: Cambridge Department of Applied Economics.

CSO: Central Statistical Office.

NBER: National Bureau of Economic Research.

UN: United Nations.

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APPENDIX B DATA

This appendix gives the 160 data series discussed in chapter 2 for the 44 quarters from 1956 Q1 to 1966 Q4. These are preceeded by a glossary of the particular notation used here, and a list identifying each series in terms of its variable and industry.

Glossary

Industries

s	sum total	i	iron, steel, metals
g	(government)	e	engineering goods
m	manufacturing	v	vehicles
mfg	(any category in m)	a	allied engineering
n	nonmanufacturing	t	textiles, clothing, footwear
f	food, drink, tobacco	p	paper, printing, publishing
c	chemicals	o	other manufacturing

Industry variables

x	output	e	earnings
l	labour	z	profits
u	unemployment	k	capital
i	investment	g	intermediate demand
s	stocks	h	materials price
p	price		

Economy variables

C	consumption	A	total indirect tax
B	consumer prices	GG	government expenditure on goods
N	exports	GL	government expenditure on labour
M	imports	GT	government transfer payments
WT	world trade	GK	government capital expenditure
RB	bank rate	V	other income
HP	hire purchase restrictions	Y	gross domestic product
TY	income tax rate	QU	first quarter seasonal
TZ	profit tax rate	QD	second quarter seasonal
TC	consumption tax rate	QT	third quarter seasonal

Redundant variables

PN export prices
PM import prices
PW world prices
PP input prices

RD debenture yield
DD rate of duty on drink and tobacco
DO rate of duty on oils

Fixed proportions

w weight
q base

a input-output coefficient

Industries are denoted by underlined lower case letters. Industry variables are lower case, economy variables upper case; one letter variables are endogenous, two letter exogenous (a few three letter are used for clarity, these always being defined in context.)

Identification of the series (number, variable, industry)

1	X	s	41	I	c	81	Z	c	121	TZ -
2	X	m	42	I	i	82	Z	i	122	TC f
3	X	n	43	I	e	83	Z	e	123	TC v
4	X	f	44	I	v	84	Z	v	124	TC t
5	X	c	45	I	a	85	Z	a	125	TC (o)
6	X	i	46	I	t	86	Z	t	126	A -
7	X	e	47	I	p	87	Z	p	127	GG -
8	X	v	48	I	o	88	Z	o	128	GL -
9	X	a	49	S	s	89	K	s	129	GT -
10	X	t	50	S	m	90	K	m	130	GK -
11	X	p	51	S	n	91	K	n	131	V -
12	X	o	52	S	f	92	K	f	132	Y -
13	L	s	53	S	c	93	K	c	133	N c
14	L	m	54	S	i	94	K	i	134	N i
15	L	n	55	S	eva	95	K	e	135	N eva
16	L	f	56	S	t	96	K	v	136	N t
17	L	c	57	S	po	97	K	a	137	N (o)
18	L	i	58	P	m	98	K	t	138	PN s
19	L	e	59	P	f	99	K	p	139	PN m
20	L	v	60	P	c	100	K	o	140	PN n
21	L	a	61	P	i	101	C	f	141	PN c
22	L	t	62	P	t	102	C	v	142	PN i
23	L	p	63	P	p	103	C	t	143	PN eva
24	L	o	64	P	(o)	104	C	(o)	144	PN t
25	U	s	65	E	s	105	B	f	145	PN (o)
26	U	m	66	E	m	106	B	v	146	PM s
27	U	n	67	E	n	107	B	t	147	PM m
28	U	f	68	E	f	108	B	(o)	148	PM n
29	U	c	69	E	c	109	N	s	149	PW s
30	U	i	70	E	i	110	N	m	150	PW m
31	U	e	71	E	e	111	N	n	151	PW n
32	U	v	72	E	v	112	M	s	152	PP m
33	U	a	73	E	a	113	M	m	153	PP f
34	U	t	74	E	t	114	M	n	154	PP c
35	U	p	75	E	p	115	WT	s	155	PP e
36	U	o	76	E	o	116	WT	m	156	PP t
37	I	s	77	Z	s	117	WT	n	157	PP p
38	I	m	78	Z	m	118	RB	-	158	RD -
39	I	n	79	Z	n	119	HP	-	159	DD -
40	I	f	80	Z	f	120	TY	-	160	DO -

SERIES 1 - 10

0.854	0.897	0.723	0.833	0.784	1.027	0.836	0.946	1.040	1.071
0.908	0.881	0.831	0.905	0.777	0.982	0.828	0.817	1.017	1.035
0.883	0.809	0.850	0.857	0.729	0.857	0.777	0.716	0.931	0.953
0.924	0.897	0.860	0.977	0.784	0.964	0.872	0.788	1.017	1.035
0.870	0.913	0.761	0.873	0.817	1.000	0.879	0.896	0.993	1.098
0.913	0.897	0.852	0.921	0.797	0.991	0.857	0.874	0.970	1.035
0.894	0.841	0.877	0.873	0.763	0.893	0.807	0.824	0.939	0.962
0.939	0.913	0.906	0.953	0.811	0.991	0.887	0.910	1.001	1.008
0.850	0.913	0.777	0.981	0.811	0.982	0.887	0.961	1.009	0.981
0.877	0.881	0.851	0.953	0.797	0.902	0.843	0.925	0.970	0.881
0.895	0.817	0.925	0.889	0.750	0.776	0.792	0.802	0.915	0.863
0.945	0.913	0.940	0.969	0.824	0.839	0.916	0.910	0.986	0.962
0.834	0.901	0.765	0.897	0.883	0.857	0.889	0.944	0.983	0.924
0.936	0.954	0.890	0.998	0.916	0.918	0.906	1.034	1.033	0.951
0.918	0.883	0.912	0.952	0.875	0.848	0.846	0.872	0.893	0.879
1.000	1.015	0.949	0.989	0.949	1.032	0.992	1.088	1.023	1.030
0.928	1.024	0.840	0.934	1.006	1.093	0.992	1.151	1.063	1.039
0.980	1.024	0.933	1.026	1.014	1.093	0.966	1.124	1.053	0.995
0.971	0.936	1.010	0.961	0.973	0.988	0.898	0.971	0.973	0.942
0.995	1.042	0.955	1.026	1.014	1.084	1.043	0.989	1.043	1.048
0.956	1.033	0.906	0.980	1.055	1.066	1.009	1.034	1.043	1.022
1.025	1.042	1.014	1.054	1.031	1.058	1.052	1.052	1.013	0.986
1.010	0.954	1.058	0.998	0.965	0.909	0.983	0.899	0.913	0.942
1.033	1.024	1.051	1.044	1.022	0.970	1.094	0.944	0.973	0.995
0.975	1.024	0.974	0.998	1.055	0.970	1.052	1.052	0.973	0.995
1.030	1.042	1.057	1.072	1.063	0.979	1.069	1.052	0.983	0.942
1.007	0.971	1.074	1.017	1.022	0.892	1.000	0.908	0.933	0.924
1.038	1.042	1.069	1.063	1.071	0.953	1.094	0.969	0.963	1.004
1.007	1.024	1.019	0.998	1.104	0.953	1.077	1.070	0.973	0.986
1.089	1.060	1.141	1.091	1.145	0.979	1.060	1.124	0.973	0.977
1.075	1.015	1.147	1.054	1.096	0.935	1.034	1.007	0.923	0.959
1.158	1.121	1.222	1.109	1.178	1.075	1.146	1.106	1.043	1.075
1.095	1.130	1.114	1.026	1.227	1.110	1.154	1.178	1.063	1.066
1.160	1.174	1.207	1.128	1.268	1.163	1.185	1.232	1.083	1.048
1.124	1.077	1.218	1.091	1.194	1.023	1.094	0.989	0.963	0.995
1.184	1.192	1.260	1.137	1.260	1.171	1.257	1.151	1.083	1.102
1.129	1.201	1.152	1.081	1.317	1.233	1.257	1.196	1.123	1.093
1.194	1.192	1.272	1.128	1.309	1.206	1.205	1.259	1.093	1.057
1.178	1.104	1.315	1.100	1.251	1.066	1.146	1.007	0.963	1.022
1.220	1.227	1.313	1.165	1.292	1.171	1.317	1.205	1.073	1.119
1.167	1.236	1.220	1.118	1.366	1.180	1.334	1.277	1.093	1.128
1.191	1.218	1.281	1.183	1.341	1.145	1.274	1.250	1.033	1.084
1.182	1.130	1.323	1.118	1.309	0.997	1.214	1.070	0.953	1.004
1.228	1.201	1.352	1.165	1.366	1.058	1.365	1.097	0.963	1.048

SERIES 11 - 20

0.881	0.841	22190	8688	13502	779	502	618	1917	868
0.841	0.895	22253	8610	13643	790	501	612	1899	859
0.759	0.850	22335	8593	13742	806	503	611	1898	844
0.861	0.904	22410	8654	13756	804	504	616	1916	843
0.881	0.877	22179	8554	13625	770	505	617	1906	821
0.868	0.895	22348	8552	13797	787	503	614	1896	834
0.793	0.859	22377	8480	13897	822	504	582	1908	851
0.875	0.895	22526	8709	13817	808	511	616	1954	861
0.888	0.850	22285	8615	13670	784	506	608	1940	863
0.881	0.859	22314	8527	13787	796	504	594	1917	859
0.814	0.823	22375	8479	13896	822	504	580	1904	849
0.888	0.922	22272	8485	13787	809	511	580	1925	844
0.917	0.843	22026	8431	13596	779	511	577	1917	847
0.917	0.937	22151	8446	13705	788	510	578	1913	856
0.815	0.920	22374	8559	13815	815	515	587	1937	859
0.985	1.005	22477	8706	13772	817	521	601	1984	876
1.036	0.979	22385	8753	13633	791	521	613	2007	896
1.019	1.014	22565	8826	13739	805	524	622	2032	906
0.926	0.971	22850	8898	13952	829	530	630	2058	911
1.070	1.014	22922	8962	13960	830	533	636	2044	903
1.078	0.971	22774	8915	13860	803	525	638	2108	884
1.036	1.031	22845	8922	13923	811	534	639	2109	896
0.926	0.988	23030	8972	14058	839	539	635	2127	897
1.027	1.031	23116	9017	14099	841	529	629	2171	887
1.036	0.979	22909	8943	13966	815	525	618	2163	886
1.070	1.048	22971	8779	14193	830	500	609	2154	882
0.959	0.996	23081	8903	14179	855	520	605	2144	876
1.087	1.031	23081	8858	14223	834	508	596	2161	863
1.044	0.937	22609	8752	13657	802	502	591	2141	861
1.095	1.065	22855	8715	14140	813	501	588	2123	860
1.010	1.048	23014	8737	14277	844	503	587	2119	857
1.197	1.150	23174	8838	14336	831	506	604	2160	866
1.180	1.133	23002	8809	14193	794	504	612	2175	866
1.223	1.209	23142	8841	14301	794	506	620	2185	869
1.078	1.150	23240	8852	14388	815	508	622	2180	868
1.223	1.243	23333	8952	14381	815	509	630	2216	873
1.240	1.201	23100	8890	14210	785	506	631	2217	874
1.214	1.226	23152	8869	14283	790	505	629	2213	871
1.104	1.167	23327	8911	14416	821	510	627	2225	867
1.257	1.243	23299	8962	14337	817	511	629	2256	870
1.299	1.226	23549	8894	14655	802	523	628	2312	859
1.282	1.226	23628	8871	14757	803	524	621	2309	855
1.129	1.201	23624	8909	14715	835	527	618	2315	845
1.189	1.218	23267	8813	14474	825	525	611	2310	825

SERIES 21 - 30

765	1590	547	931	191.2	79.4	111.8	11.2	3.5	3.9
755	1563	550	909	209.9	90.9	119.0	9.4	3.4	4.9
752	1550	555	906	222.1	102.1	120.0	8.6	3.4	5.9
762	1549	558	917	229.2	77.3	151.9	10.2	3.6	4.4
754	1571	556	898	337.0	117.2	219.8	15.9	4.6	7.9
757	1539	561	895	270.9	101.2	169.7	12.9	4.4	8.2
736	1447	574	880	211.9	76.4	135.6	10.3	4.0	4.8
764	1542	572	909	273.2	91.3	181.9	12.0	4.3	8.5
756	1522	569	897	376.0	134.5	241.5	16.5	5.5	12.5
746	1432	570	885	395.3	178.8	216.5	16.1	6.0	20.3
742	1445	575	896	366.6	167.6	199.0	15.3	6.1	18.9
740	1446	570	892	474.7	199.9	274.8	20.1	7.0	27.2
738	1440	568	883	543.5	210.7	332.8	22.7	7.4	26.4
731	1438	569	888	422.8	173.2	249.6	19.2	6.8	21.0
776	1452	576	906	332.4	126.5	205.9	15.7	6.0	10.7
769	1468	589	925	366.2	118.2	248.0	16.0	5.9	8.2
795	1472	593	929	386.8	126.6	260.2	17.2	6.1	8.0
793	1472	600	935	286.4	101.4	185.0	13.2	5.1	6.1
799	1456	605	943	242.0	87.8	154.2	10.7	4.6	5.3
804	1473	608	948	296.7	108.8	187.9	11.9	4.8	6.4
803	1470	611	937	335.9	137.7	198.2	13.1	4.9	8.4
797	1458	608	933	248.9	94.4	154.5	11.1	4.4	7.5
795	1452	613	938	225.7	81.6	144.1	9.9	4.4	7.3
800	1443	623	954	326.3	128.6	197.7	12.0	5.4	17.0
792	1445	621	942	385.5	147.6	237.9	15.6	6.3	21.5
787	1433	621	941	347.8	140.7	207.1	14.3	6.4	16.5
782	1418	624	943	343.4	137.1	206.3	14.2	6.5	13.2
775	1408	626	952	465.4	184.2	281.2	18.2	7.8	23.3
764	1401	621	933	434.5	242.9	191.6	24.2	9.1	25.0
755	1391	619	934	474.4	195.8	278.6	19.8	8.0	30.0
754	1375	623	939	377.9	144.8	233.1	15.9	7.2	11.8
766	1383	629	958	397.9	135.1	262.8	16.1	7.4	10.3
766	1379	624	953	397.7	129.5	268.2	17.3	7.3	9.6
770	1381	624	958	310.2	108.1	202.1	13.5	6.0	7.0
771	1371	623	961	267.4	92.5	174.9	11.4	6.1	6.4
786	1378	630	982	293.0	92.6	200.4	11.5	6.1	5.8
784	1341	626	973	313.8	97.1	216.7	13.2	6.2	6.5
782	1350	624	971	256.9	83.7	173.2	10.7	5.8	5.8
782	1344	630	973	255.2	95.3	159.9	9.4	5.5	5.2
791	1345	634	976	269.9	81.1	188.8	9.8	5.3	6.0
800	1350	640	980	290.9	87.1	203.8	10.9	5.3	7.6
796	1345	640	978	235.7	78.1	157.6	9.3	4.9	7.0
792	1344	647	982	236.5	78.9	157.6	8.8	5.3	6.0
787	1318	643	969	484.3	219.8	264.5	13.4	6.9	17.6

SERIES 31 - 40

12.6	3.7	8.4	18.1	2.9	15.1	563	196	367	21.8
13.5	8.8	8.3	25.5	2.8	14.2	577	201	376	22.4
17.7	23.3	8.9	21.1	2.5	10.6	583	214	369	21.9
13.1	5.5	9.3	16.8	3.0	11.3	649	252	397	25.0
18.8	11.5	14.3	19.1	4.6	20.5	616	217	399	22.2
17.7	7.6	13.1	18.5	3.3	15.5	631	221	410	20.1
13.5	5.0	9.1	15.9	2.7	10.9	653	221	432	22.6
15.3	6.0	10.6	19.4	3.3	11.8	705	244	461	25.7
23.6	7.3	14.9	30.4	4.6	19.2	674	216	458	25.7
25.0	13.0	17.0	56.2	4.7	20.4	670	221	449	25.3
25.0	8.5	17.1	54.3	4.5	17.9	679	223	456	26.1
34.2	10.4	22.6	51.8	5.3	21.2	713	236	477	25.3
37.2	10.5	22.3	50.2	6.6	27.4	692	208	484	24.8
30.9	8.3	23.4	36.9	5.4	21.2	686	209	477	27.9
19.9	7.2	20.5	25.0	4.8	16.7	715	213	502	27.3
19.8	5.9	21.0	22.2	4.4	14.8	782	244	538	26.5
22.2	6.3	20.1	24.3	4.6	17.5	773	223	550	28.9
15.9	4.4	17.8	22.1	3.6	13.1	780	238	542	31.4
15.2	4.2	15.0	18.1	3.0	11.5	807	273	534	33.5
15.8	16.4	20.3	16.7	3.6	12.8	882	295	587	34.3
19.2	35.0	18.2	17.7	3.9	17.4	877	273	604	33.3
15.7	9.5	15.6	15.9	3.6	11.1	895	305	590	36.0
13.5	4.2	14.1	15.1	3.0	10.3	912	331	581	34.3
17.0	19.6	19.1	21.5	4.0	12.9	967	331	636	36.6
21.9	9.7	21.4	28.0	5.0	18.2	892	286	606	34.2
22.2	7.9	21.1	30.3	5.1	16.9	903	294	609	35.3
24.5	8.0	20.2	29.7	4.7	16.2	916	290	626	34.0
30.7	17.6	31.1	30.5	5.8	19.2	936	298	638	32.8
40.9	18.7	39.6	39.7	7.6	38.0	821	241	580	32.2
36.3	10.7	33.3	35.9	6.8	22.9	910	266	644	36.8
27.3	9.0	24.3	26.0	5.3	18.0	949	267	682	36.7
25.6	8.8	24.3	21.2	5.2	16.4	1036	293	743	41.6
23.2	7.0	22.5	20.1	5.6	16.8	1057	273	784	40.5
19.6	7.6	18.5	17.3	4.5	13.3	1054	304	750	43.9
17.8	5.8	14.3	15.0	3.9	11.9	1109	311	798	45.4
16.9	8.7	13.5	14.6	3.8	11.6	1201	356	845	44.1
18.8	5.6	13.5	16.4	4.3	12.8	1212	334	878	42.5
15.9	5.1	11.4	14.2	3.7	11.0	1133	329	804	42.4
15.1	22.7	10.7	12.3	3.6	11.0	1159	352	807	46.6
15.7	5.9	11.8	11.7	3.8	11.1	1274	387	887	46.6
16.6	5.9	13.0	11.8	3.9	12.0	1240	347	893	40.9
14.7	7.2	10.3	10.7	3.5	10.6	1169	346	823	40.2
15.6	7.9	10.4	10.7	3.5	10.8	1229	366	863	41.8
33.7	70.7	21.6	26.6	6.0	23.3	1294	397	897	43.3

SERIES 41 - 50

29.4	21.1	32.7	20.9	9.6	19.4	13.6	16.2	8669	4479
32.8	26.3	33.9	21.4	9.9	16.1	15.0	14.0	8695	4588
34.4	27.0	36.2	23.0	10.6	14.9	15.0	18.7	8721	4643
44.0	26.3	46.1	30.5	13.5	16.8	20.6	17.6	8797	4694
45.4	30.3	35.6	20.1	10.9	14.1	17.6	16.5	8866	4849
48.0	34.3	31.5	23.5	9.7	17.0	18.6	16.2	8875	4899
47.7	32.7	36.0	19.0	11.1	13.0	16.7	18.6	8924	4978
54.5	31.7	43.9	19.8	13.5	12.7	19.6	16.2	8933	4941
47.5	30.5	33.7	13.5	10.9	9.3	20.7	14.4	8915	4983
46.7	31.9	36.1	15.0	11.6	11.4	21.3	14.3	8917	4993
44.5	38.5	34.9	15.3	11.3	10.9	16.3	15.4	8909	4972
51.4	29.0	41.5	18.1	13.4	13.0	20.8	16.4	8931	4945
50.2	28.1	32.6	16.2	12.7	12.5	15.1	15.1	8929	4949
43.5	30.2	33.1	15.6	12.9	15.3	15.0	15.9	8921	4952
34.3	41.7	33.8	14.9	13.2	16.8	12.9	17.6	8939	4980
44.6	30.3	45.1	21.3	17.5	22.1	16.8	19.6	8991	5048
37.5	35.8	37.8	13.9	12.3	18.9	17.1	20.9	9276	5193
37.1	40.8	39.0	18.2	12.7	22.3	14.8	21.2	9496	5347
38.3	64.7	43.1	22.7	14.1	19.8	15.2	21.7	9595	5449
43.8	47.4	54.5	24.1	17.7	24.8	20.8	27.1	9698	5590
41.9	56.2	42.1	23.4	14.0	22.3	17.0	22.7	9883	5678
47.0	63.0	46.4	25.1	15.4	26.9	17.4	27.3	10097	5794
48.6	78.9	48.8	30.4	16.2	29.1	19.5	25.2	10152	5838
53.2	55.4	59.0	30.2	19.5	27.4	22.8	27.0	10105	5877
47.2	55.0	42.7	24.9	13.9	24.5	19.5	24.3	10263	5930
49.0	61.1	42.1	22.7	13.7	24.3	19.3	26.6	10284	5952
43.2	58.7	44.4	28.5	14.4	21.6	18.4	27.1	10272	5922
47.9	47.7	49.3	24.8	16.0	22.7	21.6	28.8	10288	5973
37.6	33.9	42.0	22.4	12.3	20.6	18.1	21.8	10255	6006
44.0	35.1	44.2	24.5	12.9	21.9	19.9	26.6	10505	6091
41.9	31.9	46.9	26.4	13.7	22.6	19.6	27.6	10344	5967
48.4	26.1	52.5	26.7	15.3	26.3	21.5	34.9	10702	6210
48.9	24.9	47.6	20.9	12.7	28.7	20.6	28.4	10901	6342
52.8	28.0	52.1	23.1	13.9	34.9	20.1	34.7	11153	6504
51.3	33.7	54.3	20.1	14.5	34.3	22.6	34.5	11435	6646
67.0	27.7	65.0	26.1	17.4	41.6	25.6	41.6	11665	6806
58.9	26.8	62.9	24.7	16.0	36.4	27.5	37.8	11816	6888
60.7	29.9	59.3	35.2	15.1	33.1	27.1	36.1	12066	7054
65.6	35.6	60.2	27.6	15.4	36.5	23.6	41.0	12203	7112
71.8	28.3	77.1	32.7	19.6	36.2	27.5	47.2	12294	7199
72.8	29.5	60.1	28.1	15.4	33.9	22.9	42.9	12589	7371
70.0	31.3	63.2	27.2	16.2	35.7	23.0	39.5	12873	7468
75.3	39.7	63.7	25.6	16.4	33.3	23.3	47.3	12987	7562
86.1	35.3	74.6	28.9	19.2	33.8	29.2	46.4	12790	7486

SERIES 51 - 60

4190	688	400	332	1984	631	442	0.9188	0.9596	0.9739
4107	696	410	342	2044	641	453	0.9188	0.9596	0.9739
4078	687	422	350	2091	625	466	0.9188	0.9596	0.9739
4103	677	424	355	2119	649	468	0.9188	0.9596	0.9739
4017	767	429	360	2154	665	472	0.9462	1.0049	1.0127
3976	741	439	367	2190	675	485	0.9462	0.9904	1.0023
3946	732	447	379	2262	664	492	0.9445	0.9460	1.0051
3992	701	451	377	2250	663	498	0.9524	0.9315	1.0051
3932	715	447	382	2282	665	492	0.9524	0.9296	1.0032
3924	713	446	386	2307	649	492	0.9551	0.9496	0.9994
3937	715	444	388	2319	617	489	0.9551	0.9532	0.9919
3986	710	446	384	2292	617	493	0.9587	0.9659	0.9994
3980	728	440	383	2290	621	486	0.9605	0.9723	1.0042
3969	729	439	382	2284	633	485	0.9569	0.9632	1.0145
3959	737	440	385	2302	631	485	0.9577	0.9695	1.0013
3943	720	448	393	2348	645	494	0.9605	0.9777	1.0079
4083	756	451	399	2359	640	489	0.9622	0.9705	1.0108
4149	761	465	413	2429	665	507	0.9720	0.9650	1.0060
4146	770	480	452	2480	664	521	0.9764	0.9741	0.9985
4109	770	494	460	2556	670	548	0.9790	0.9723	0.9928
4205	783	495	469	2583	697	575	0.9862	0.9650	0.9947
4303	788	502	472	2629	724	595	0.9941	0.9759	0.9966
4313	793	508	480	2670	693	597	1.0065	0.9723	0.9938
4227	781	507	475	2733	691	592	1.0101	0.9786	0.9919
4334	821	492	461	2744	705	586	1.0172	1.0004	0.9938
4332	833	488	462	2809	714	579	1.0190	1.0022	0.9890
4350	839	498	462	2823	681	568	1.0235	1.0085	0.9853
4315	846	509	462	2826	683	573	1.0261	1.0158	0.9815
4248	880	508	462	2842	696	598	1.0320	1.0255	0.9824
4414	936	507	461	2823	707	587	1.0380	1.0512	0.9872
4377	862	510	463	2829	680	582	1.0328	1.0185	0.9881
4492	946	525	468	2903	688	581	1.0415	1.0548	0.9928
4559	949	524	483	2968	730	592	1.0502	1.0675	0.9985
4649	952	541	505	3059	759	602	1.0683	1.0621	1.0060
4790	974	557	509	3140	733	611	1.0769	1.0738	1.0117
4858	986	577	517	3229	740	624	1.0872	1.0847	1.0202
4928	977	562	529	3289	703	660	1.1011	1.0956	1.0344
5012	956	602	529	3390	712	674	1.1218	1.0902	1.0372
5091	973	604	544	3445	674	689	1.1261	1.0875	1.0325
5095	948	610	558	3526	677	683	1.1305	1.0929	1.0316
5218	996	610	562	3608	704	707	1.1362	1.1110	1.0391
5406	996	623	544	3735	731	721	1.1494	1.1201	1.0382
5425	1000	632	593	3765	715	727	1.1581	1.1210	1.0419
5304	981	626	564	3779	701	720	1.1563	1.1201	1.0382

SERIES 61 - 70

0.8720	0.9550	0.9839	0.8886	0.128	0.132	0.125	0.112	0.130	0.158
0.8720	0.9550	0.9839	0.8886	0.115	0.123	0.109	0.103	0.141	0.152
0.8720	0.9550	0.9839	0.8886	0.115	0.122	0.110	0.102	0.138	0.152
0.8720	0.9550	0.9839	0.8886	0.114	0.125	0.108	0.105	0.143	0.154
0.9354	0.9694	0.9884	0.9098	0.131	0.136	0.126	0.116	0.139	0.165
0.9393	0.9809	0.9929	0.9109	0.121	0.134	0.114	0.112	0.149	0.164
1.0042	0.9857	0.9965	0.9035	0.119	0.128	0.113	0.110	0.147	0.166
1.0104	0.9809	0.9965	0.9230	0.126	0.140	0.118	0.116	0.152	0.173
1.0119	0.9752	0.9920	0.9256	0.130	0.140	0.123	0.126	0.153	0.170
1.0019	0.9665	0.9884	0.9315	0.128	0.140	0.121	0.121	0.155	0.173
1.0019	0.9617	0.9875	0.9334	0.127	0.138	0.120	0.121	0.153	0.173
1.0019	0.9522	0.9857	0.9389	0.130	0.142	0.123	0.122	0.159	0.173
1.0027	0.9416	0.9785	0.9437	0.140	0.151	0.133	0.132	0.158	0.181
1.0027	0.9464	0.9668	0.9374	0.133	0.145	0.126	0.128	0.162	0.177
0.9918	0.9502	0.9650	0.9413	0.131	0.140	0.125	0.127	0.159	0.178
0.9942	0.9637	0.9659	0.9397	0.136	0.151	0.128	0.131	0.167	0.185
0.9918	0.9742	0.9650	0.9423	0.152	0.160	0.146	0.143	0.159	0.192
0.9918	0.9857	0.9704	0.9604	0.139	0.151	0.132	0.132	0.172	0.187
0.9949	0.9895	0.9677	0.9674	0.137	0.148	0.129	0.130	0.166	0.188
0.9942	1.0001	0.9749	0.9705	0.143	0.154	0.136	0.136	0.179	0.191
0.9965	1.0097	0.9794	0.9827	0.161	0.169	0.156	0.150	0.166	0.200
0.9973	1.0164	0.9866	0.9929	0.150	0.160	0.143	0.142	0.180	0.196
1.0065	1.0135	0.9929	1.0173	0.148	0.158	0.142	0.140	0.179	0.196
1.0073	1.0116	0.9963	1.0229	0.152	0.161	0.147	0.146	0.180	0.194
1.0127	1.0078	1.0010	1.0313	0.165	0.172	0.161	0.157	0.179	0.201
1.0351	1.0097	1.0001	1.0307	0.158	0.168	0.152	0.150	0.191	0.200
1.0344	1.0087	1.0001	1.0393	0.157	0.166	0.151	0.149	0.191	0.201
1.0344	1.0097	1.0010	1.0432	0.160	0.169	0.155	0.152	0.191	0.204
1.0351	1.0126	1.0028	1.0508	0.162	0.171	0.157	0.150	0.204	0.206
1.0336	1.0174	1.0028	1.0557	0.170	0.177	0.165	0.158	0.199	0.211
1.0336	1.0193	1.0073	1.0519	0.169	0.176	0.164	0.157	0.197	0.213
1.0344	1.0240	1.0100	1.0574	0.172	0.180	0.167	0.157	0.202	0.220
1.0375	1.0404	1.0226	1.0647	0.176	0.187	0.169	0.168	0.214	0.228
1.0375	1.0471	1.0271	1.0986	0.179	0.189	0.172	0.174	0.209	0.230
1.0390	1.0538	1.0298	1.1097	0.181	0.189	0.175	0.175	0.206	0.231
1.0405	1.0595	1.0416	1.1229	0.185	0.195	0.178	0.178	0.217	0.238
1.0467	1.0643	1.0677	1.1393	0.188	0.203	0.178	0.183	0.239	0.246
1.0583	1.0720	1.0695	1.1772	0.194	0.209	0.185	0.190	0.231	0.251
1.0606	1.0749	1.0686	1.1863	0.191	0.204	0.182	0.189	0.224	0.251
1.0599	1.0826	1.0731	1.1916	0.197	0.212	0.187	0.195	0.231	0.259
1.0606	1.0864	1.0812	1.1992	0.198	0.219	0.185	0.200	0.257	0.259
1.0954	1.0969	1.0857	1.2099	0.203	0.224	0.191	0.206	0.241	0.263
1.0985	1.1046	1.0857	1.2239	0.201	0.219	0.190	0.203	0.238	0.263
1.0978	1.1008	1.0866	1.2223	0.202	0.221	0.191	0.207	0.247	0.260

SERIES 71 - 80

0.147	0.142	0.148	0.100	0.164	0.125	732	446	286	61
0.127	0.156	0.142	0.099	0.140	0.120	802	473	329	71
0.124	0.152	0.143	0.100	0.115	0.119	831	505	326	87
0.127	0.154	0.143	0.103	0.147	0.123	929	542	387	88
0.143	0.145	0.149	0.109	0.155	0.132	760	462	298	68
0.143	0.155	0.149	0.108	0.153	0.130	826	518	308	79
0.126	0.174	0.150	0.108	0.139	0.128	908	533	375	85
0.150	0.164	0.156	0.110	0.157	0.134	931	564	367	87
0.147	0.165	0.153	0.111	0.170	0.132	718	441	277	72
0.148	0.169	0.154	0.111	0.157	0.134	848	515	333	84
0.144	0.169	0.154	0.111	0.147	0.133	886	521	365	90
0.150	0.169	0.156	0.112	0.159	0.139	898	546	352	93
0.160	0.149	0.155	0.115	0.178	0.142	776	454	322	75
0.153	0.171	0.153	0.114	0.171	0.140	906	549	357	89
0.143	0.205	0.153	0.114	0.129	0.138	995	607	388	87
0.156	0.188	0.153	0.117	0.183	0.146	1062	652	410	108
0.175	0.169	0.159	0.122	0.184	0.157	927	533	394	87
0.157	0.193	0.154	0.120	0.173	0.147	1104	648	456	96
0.145	0.133	0.154	0.120	0.156	0.146	1141	655	486	92
0.162	0.188	0.157	0.122	0.178	0.151	1140	673	467	104
0.185	0.172	0.168	0.131	0.191	0.166	1014	549	465	97
0.163	0.205	0.164	0.128	0.185	0.159	1056	594	462	95
0.161	0.195	0.163	0.128	0.162	0.157	1137	606	531	102
0.164	0.200	0.163	0.129	0.191	0.163	1115	605	510	110
0.176	0.183	0.171	0.132	0.206	0.173	1061	570	491	102
0.173	0.208	0.168	0.131	0.190	0.168	1009	547	462	103
0.170	0.201	0.168	0.131	0.175	0.166	1126	602	524	105
0.174	0.204	0.169	0.134	0.195	0.172	1184	619	565	115
0.174	0.206	0.169	0.136	0.194	0.169	1195	607	588	112
0.181	0.212	0.173	0.140	0.203	0.180	1175	613	562	114
0.177	0.211	0.174	0.140	0.198	0.179	1274	680	594	119
0.183	0.216	0.179	0.143	0.204	0.184	1391	736	655	131
0.190	0.229	0.185	0.147	0.209	0.188	1282	698	584	112
0.192	0.225	0.184	0.147	0.217	0.192	1368	701	667	115
0.190	0.228	0.189	0.149	0.214	0.195	1424	750	674	114
0.197	0.234	0.194	0.154	0.221	0.200	1506	806	700	124
0.206	0.238	0.202	0.157	0.230	0.203	1446	814	632	127
0.215	0.248	0.208	0.161	0.237	0.211	1420	748	672	128
0.206	0.240	0.207	0.160	0.230	0.208	1469	760	709	130
0.217	0.248	0.212	0.166	0.242	0.214	1527	817	710	132
0.216	0.255	0.214	0.168	0.246	0.215	1419	792	627	130
0.223	0.244	0.220	0.170	0.253	0.222	1363	728	635	133
0.218	0.254	0.217	0.171	0.246	0.219	1346	727	619	141
0.220	0.243	0.219	0.172	0.250	0.221	1480	790	690	136

SERIES 81 - 90

52	56	96	34	32	55	29	31	40698	13062
66	52	94	38	39	43	32	38	40963	13210
58	60	98	17	38	69	43	35	41228	13358
67	63	121	44	36	54	33	36	41493	13505
50	55	103	34	36	55	29	32	41783	13661
70	56	104	45	43	51	33	37	42074	13817
70	72	110	30	32	60	38	36	42364	13973
75	71	131	46	35	46	34	39	42655	14128
47	55	97	39	31	40	28	32	42945	14269
66	52	101	55	44	40	39	34	43235	14410
69	65	109	33	33	50	36	36	43525	14551
67	63	121	49	36	45	35	37	43815	14692
46	57	99	36	30	45	31	35	44103	14818
67	65	105	53	39	51	43	37	44392	14943
83	82	120	40	39	66	40	50	44681	15069
86	77	127	74	27	69	39	45	44970	15194
60	60	110	42	37	54	37	46	45396	15359
79	84	117	56	47	73	46	50	45822	15523
37	102	120	44	36	70	53	51	46247	15687
93	88	131	63	33	62	47	52	46673	15851
67	62	111	37	40	52	38	45	47151	16055
74	79	118	33	39	67	41	48	47629	16260
80	71	126	39	35	62	42	49	48107	16464
79	70	130	39	32	54	40	51	48584	16668
73	47	114	46	45	54	44	45	48974	16850
68	66	117	29	24	58	41	41	49365	17031
80	54	131	51	41	52	39	49	49756	17213
84	60	133	43	21	65	46	52	50147	17394
77	49	117	51	47	58	47	49	50579	17538
73	61	134	48	23	69	42	49	51010	17683
86	59	145	72	32	71	42	54	51442	17827
96	77	141	56	33	89	53	60	51874	17972
89	67	134	55	70	60	49	62	52369	18144
85	77	159	53	24	67	52	69	52865	18316
99	79	172	82	23	66	45	70	53360	18488
108	86	162	60	47	84	58	77	53855	18661
99	94	154	58	88	71	55	68	54427	18866
96	75	172	50	28	73	56	70	54999	19072
105	67	166	84	25	64	49	70	55571	19278
112	80	163	62	43	87	63	75	56143	19483
99	93	157	48	63	71	61	70	56746	19707
92	78	161	48	30	71	48	67	57349	19931
95	57	159	58	39	61	49	68	57952	20155
101	69	164	43	56	80	65	76	58555	20379

SERIES 91 - 100

27636	1593.0	1489.3	1384.2	2094.0	1241.8	759.6	2559.1	982.8	958.7
27753	1606.6	1523.6	1410.0	2117.5	1257.8	767.4	2561.8	993.8	971.6
27870	1620.1	1557.9	1435.8	2140.9	1273.8	775.2	2564.5	1004.7	984.5
27988	1633.7	1592.3	1461.6	2164.3	1289.8	783.0	2567.1	1015.7	997.5
28122	1648.2	1631.0	1489.7	2187.0	1305.3	791.5	2569.6	1027.6	1011.1
28257	1662.7	1669.6	1517.8	2209.6	1320.7	800.0	2572.1	1039.5	1024.7
28392	1677.2	1708.3	1545.9	2232.2	1336.2	808.4	2574.6	1051.4	1038.3
28526	1691.7	1746.9	1574.0	2254.9	1351.6	816.9	2577.1	1063.2	1052.0
28676	1706.3	1784.4	1601.8	2276.6	1362.2	825.3	2575.4	1072.3	1065.0
28825	1721.0	1822.0	1629.7	2298.4	1372.8	833.7	2573.6	1081.3	1078.1
28974	1735.6	1859.5	1657.5	2320.1	1383.3	842.0	2571.8	1090.4	1091.2
29122	1750.3	1897.0	1685.4	2341.8	1393.9	850.4	2570.0	1099.4	1104.3
29285	1763.4	1928.0	1710.0	2359.7	1402.7	860.9	2568.3	1106.5	1118.5
29448	1776.5	1959.1	1734.5	2377.5	1411.5	871.4	2566.5	1113.6	1132.7
29612	1789.6	1990.1	1759.1	2395.4	1420.3	881.9	2564.8	1120.7	1146.9
29776	1802.7	2021.2	1783.7	2413.2	1429.1	892.4	2563.1	1127.7	1161.1
30037	1819.3	2047.6	1824.9	2438.3	1441.2	902.8	2566.4	1137.7	1180.2
30299	1835.9	2074.1	1866.2	2463.5	1453.2	913.1	2569.7	1147.6	1199.3
30560	1852.5	2100.5	1907.4	2488.6	1465.3	923.5	2573.1	1157.5	1218.4
30822	1869.2	2126.9	1948.7	2513.7	1477.3	933.8	2576.4	1167.4	1237.5
31096	1886.3	2158.8	2009.4	2541.4	1496.7	945.3	2582.3	1177.7	1257.4
31369	1903.4	2190.7	2070.1	2569.1	1516.0	956.7	2588.2	1188.1	1277.3
31643	1920.5	2222.5	2130.8	2596.8	1535.4	968.2	2594.1	1198.5	1297.2
31916	1937.6	2254.4	2191.5	2624.4	1554.8	979.6	2600.0	1208.8	1317.2
32124	1954.7	2284.6	2239.5	2648.8	1572.4	989.9	2602.6	1219.5	1337.7
32334	1971.8	2314.8	2287.5	2673.2	1590.0	1000.3	2605.1	1230.1	1358.3
32543	1989.0	2345.1	2335.5	2697.5	1607.6	1010.6	2607.7	1240.7	1378.8
32753	2006.1	2375.3	2383.5	2721.9	1625.3	1021.0	2610.3	1251.3	1399.4
33040	2025.3	2396.2	2409.5	2746.9	1641.8	1029.3	2610.2	1261.2	1418.0
33328	2044.5	2417.2	2435.4	2771.9	1658.4	1037.6	2610.2	1271.1	1436.6
33615	2063.7	2438.1	2461.3	2796.9	1674.9	1045.8	2610.2	1281.0	1455.3
33902	2082.9	2459.1	2487.2	2821.8	1691.4	1054.1	2610.1	1290.9	1473.9
34225	2105.8	2488.6	2508.8	2850.5	1705.2	1065.0	2619.8	1303.2	1497.0
34549	2128.6	2518.1	2530.5	2879.2	1718.9	1075.8	2629.5	1315.4	1520.0
34872	2151.5	2547.7	2552.1	2907.9	1732.6	1086.7	2639.2	1327.6	1543.0
35195	2174.4	2577.2	2573.7	2936.6	1746.3	1097.5	2648.9	1339.9	1566.1
35561	2200.9	2618.2	2594.3	2963.4	1768.0	1108.2	2660.8	1354.6	1597.9
35927	2227.3	2659.2	2614.9	2990.1	1789.7	1118.8	2672.7	1369.4	1629.7
36294	2253.8	2700.3	2635.5	3016.9	1811.3	1129.5	2684.6	1384.1	1661.5
36660	2280.3	2741.3	2656.0	3043.6	1833.0	1140.2	2696.5	1398.9	1693.3
37039	2302.8	2800.3	2677.4	3085.2	1850.4	1147.1	2706.2	1412.7	1724.9
37418	2325.3	2859.2	2698.9	3126.8	1867.8	1154.0	2715.9	1426.5	1756.4
37797	2347.8	2918.2	2720.3	3168.5	1885.2	1160.9	2725.5	1440.2	1788.0
38177	2370.3	2977.2	2741.7	3210.1	1902.6	1167.9	2735.2	1454.0	1819.6

SERIES 101 - 110

1453	74	282	1339	0.8920	0.954	0.9427	0.8255	774.9	638.5
1548	80	336	1401	0.9297	0.954	0.9446	0.8327	816.0	682.6
1548	56	320	1491	0.9153	0.964	0.9464	0.8462	741.6	609.6
1643	39	401	1531	0.9216	0.964	0.9512	0.8538	839.1	677.6
1484	49	294	1427	0.9279	1.030	0.9559	0.8712	830.4	690.2
1585	90	254	1580	0.9762	1.011	0.9597	0.8292	642.7	707.6
1639	81	328	1563	0.9511	1.021	0.9624	0.8855	789.6	662.5
1733	65	406	1632	0.9431	1.021	0.9690	0.9183	832.2	687.1
1549	97	300	1550	0.9377	1.011	0.9718	0.9220	813.9	690.1
1655	111	364	1595	0.9664	1.011	0.9700	0.9232	768.3	654.7
1679	87	341	1678	0.9458	1.011	0.9662	0.9295	777.0	658.4
1773	89	445	1758	0.9654	1.002	0.9662	0.9391	816.9	681.0
1579	108	304	1641	0.9699	1.011	0.9615	0.9453	791.7	673.6
1690	137	382	1702	0.9538	1.021	0.9624	0.9350	845.4	730.0
1717	105	346	1768	0.9627	1.021	0.9643	0.9320	790.5	674.4
1812	125	473	1826	0.9556	1.021	0.9700	0.9499	902.4	759.4
1613	173	323	1796	0.9503	1.021	0.9737	0.9510	924.6	788.0
1744	190	418	1820	0.9564	1.021	0.9765	0.9517	904.2	783.7
1747	137	364	1921	0.9511	1.011	0.9775	0.9612	818.4	679.8
1821	69	507	1956	0.9592	0.982	0.9831	0.9830	907.8	756.9
1681	145	348	1877	0.9583	0.982	0.9878	0.9905	938.8	793.8
1810	179	417	1913	0.9725	1.021	0.9916	0.9986	938.9	793.8
1861	121	392	2038	0.9986	1.011	0.9954	1.0149	883.1	738.2
1905	76	524	2049	1.0004	1.011	1.0010	1.0359	948.2	772.5
1755	136	341	2087	1.0047	1.011	1.0039	1.0419	929.6	779.1
1897	180	434	2194	1.0379	1.022	1.0259	1.0553	985.4	834.1
1896	127	422	2285	1.0198	1.001	1.0289	1.0573	911.0	751.7
1987	98	547	2304	1.0138	0.971	1.0340	1.0674	985.7	804.2
1790	145	351	2195	1.0530	0.981	1.0360	1.0721	1014.4	844.8
1954	209	445	2326	1.0540	0.991	1.0380	1.0793	1042.8	870.6
1965	152	449	2452	1.0258	0.981	1.0380	1.0810	976.4	808.5
2051	123	589	2467	1.0389	0.981	1.0410	1.0917	1092.1	889.7
1886	232	382	2437	1.0480	0.971	1.0450	1.1022	1092.1	917.3
2040	271	472	2545	1.0851	0.981	1.0500	1.1201	1122.0	937.8
2107	207	451	2657	1.0952	0.971	1.0540	1.1247	1004.5	831.9
2185	144	614	2704	1.1002	0.991	1.0611	1.1391	1142.4	932.0
1955	263	410	2636	1.1073	0.991	1.0670	1.1483	1113.3	932.4
2144	242	500	2782	1.1505	1.031	1.0711	1.1713	1182.9	1006.2
2207	182	486	2919	1.1526	1.022	1.0751	1.1756	1133.7	941.3
2310	114	638	2956	1.1526	1.022	1.0801	1.1900	1244.0	1033.8
2087	256	430	2844	1.1576	1.031	1.0872	1.2022	1256.6	1052.5
2266	285	514	3004	1.1927	1.031	1.0971	1.2260	1206.5	1011.3
2301	160	490	3094	1.1897	1.031	1.1102	1.2279	1196.1	1011.9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

SERIES 111 - 120

111	112	113	114	115	116	117	118	119	120
136.4	984.9	238.2	746.7	0.734	0.701	0.767	0.0533	0.39	0.0651
133.4	991.8	234.3	757.5	0.786	0.780	0.793	0.0550	0.50	0.0651
132.0	936.9	206.7	730.2	0.763	0.732	0.796	0.0550	0.50	0.0651
161.5	972.6	226.5	746.1	0.833	0.792	0.876	0.0550	0.45	0.0651
140.2	1054.5	231.3	823.2	0.810	0.792	0.828	0.0517	0.35	0.0707
135.1	1047.3	231.9	815.4	0.821	0.816	0.827	0.0517	0.34	0.0707
127.1	990.9	226.2	764.7	0.798	0.780	0.817	0.0567	0.33	0.0707
145.1	972.9	233.4	739.5	0.839	0.816	0.863	0.0700	0.33	0.0707
123.8	928.8	260.1	668.7	0.786	0.768	0.806	0.0667	0.33	0.0742
113.6	900.6	244.8	655.8	0.798	0.780	0.817	0.0500	0.33	0.0742
118.6	938.1	259.8	678.3	0.786	0.756	0.818	0.0467	0.28	0.0742
135.9	980.4	269.4	711.0	0.862	0.840	0.886	0.0417	0.00	0.0742
118.1	940.8	226.8	714.0	0.798	0.762	0.836	0.0400	0.00	0.0732
115.4	983.4	259.5	723.9	0.874	0.852	0.897	0.0400	0.00	0.0732
116.1	983.7	270.0	713.7	0.857	0.828	0.886	0.0400	0.00	0.0732
143.0	1082.1	294.9	787.2	0.944	0.931	0.959	0.0400	0.00	0.0732
136.6	1125.3	345.6	779.7	0.933	0.919	0.947	0.0500	0.00	0.0774
120.5	1141.2	379.8	761.4	0.950	0.955	0.945	0.0533	0.10	0.0774
138.6	1119.3	361.2	758.1	0.927	0.901	0.954	0.0600	0.10	0.0774
150.9	1155.0	358.5	796.5	1.003	0.998	1.009	0.0533	0.10	0.0774
145.0	1154.0	364.5	789.4	0.974	0.955	0.993	0.0500	0.10	0.0815
145.1	1132.2	357.5	774.6	1.000	0.997	1.003	0.0500	0.10	0.0815
145.0	1055.0	336.3	718.7	0.976	0.957	0.995	0.0700	0.10	0.0815
175.7	1088.0	343.4	744.6	1.048	1.045	1.052	0.0617	0.10	0.0815
150.5	1132.0	353.9	778.0	1.024	1.013	1.036	0.0575	0.10	0.0863
151.3	1132.0	360.9	771.1	1.072	1.068	1.077	0.0450	0.10	0.0863
159.3	1121.0	361.1	759.9	1.024	1.013	1.036	0.0450	0.10	0.0863
181.6	1143.0	378.8	764.2	1.105	1.108	1.101	0.0450	0.10	0.0863
169.5	1154.6	375.2	779.4	1.048	1.052	1.044	0.0400	0.10	0.0821
172.3	1199.9	396.5	803.4	1.145	1.163	1.126	0.0400	0.10	0.0821
167.8	1211.2	389.4	821.8	1.113	1.124	1.101	0.0400	0.10	0.0821
202.4	1304.0	432.6	871.4	1.226	1.258	1.191	0.0400	0.10	0.0821
174.8	1399.4	491.0	908.4	1.193	1.219	1.167	0.0467	0.10	0.0855
184.2	1409.6	521.3	888.3	1.250	1.298	1.199	0.0500	0.10	0.0855
172.6	1351.3	494.1	857.2	1.201	1.227	1.175	0.0500	0.10	0.0855
210.4	1422.9	511.0	911.8	1.338	1.393	1.281	0.0633	0.10	0.0855
180.9	1364.1	484.5	879.5	1.250	1.314	1.182	0.0700	0.10	0.0975
176.7	1422.9	537.5	885.3	1.363	1.432	1.289	0.0667	0.11	0.0975
192.4	1397.9	523.4	874.5	1.306	1.345	1.265	0.0600	0.13	0.0975
210.2	1446.4	543.6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

SERIES 121 - 130

0.2520	0.1541	0.2127	0.0399	0.1573	605	391	450	298	184
0.2520	0.1541	0.2127	0.0399	0.1601	603	403	462	301	175
0.2520	0.1541	0.2127	0.0399	0.1592	617	411	473	289	187
0.2520	0.1541	0.2127	0.0399	0.1648	642	419	481	309	202
0.2657	0.1470	0.2250	0.0382	0.1615	646	405	500	310	186
0.2657	0.1470	0.2250	0.0382	0.1549	621	401	496	314	176
0.2657	0.1470	0.2250	0.0382	0.1605	628	398	492	304	187
0.2657	0.1470	0.2250	0.0382	0.1656	653	404	500	329	210
0.2809	0.1496	0.2329	0.0371	0.1515	669	412	526	345	202
0.2809	0.1496	0.2329	0.0371	0.1543	645	406	518	372	182
0.2809	0.1496	0.2329	0.0371	0.1535	660	405	517	373	185
0.2809	0.1496	0.2329	0.0371	0.1588	664	415	531	394	203
0.2383	0.1499	0.2134	0.0369	0.1558	723	432	547	394	196
0.2383	0.1499	0.2134	0.0369	0.1589	675	438	556	430	183
0.2383	0.1499	0.2134	0.0369	0.1573	711	435	550	396	206
0.2383	0.1499	0.2134	0.0369	0.1635	706	441	559	417	233
0.1448	0.1537	0.2148	0.0383	0.1323	721	464	587	407	219
0.1448	0.1537	0.2148	0.0383	0.1355	709	452	573	419	183
0.1448	0.1537	0.2148	0.0383	0.1360	717	442	560	408	213
0.1448	0.1537	0.2148	0.0383	0.1435	741	487	618	419	226
0.1606	0.1495	0.2155	0.0404	0.1441	735	533	645	426	235
0.1606	0.1495	0.2155	0.0404	0.1457	749	501	605	454	212
0.1606	0.1495	0.2155	0.0404	0.1460	743	508	614	448	256
0.1606	0.1495	0.2155	0.0404	0.1538	815	527	637	473	235
0.1941	0.1483	0.1863	0.0579	0.1507	759	521	659	495	271
0.1941	0.1483	0.1863	0.0579	0.1528	826	531	672	484	239
0.1941	0.1483	0.1863	0.0579	0.1529	813	531	672	471	260
0.1941	0.1483	0.1863	0.0579	0.1579	871	540	684	529	265
0.1481	0.1515	0.1228	0.0622	0.1526	780	543	700	549	256
0.1481	0.1515	0.1228	0.0622	0.1555	849	557	720	545	255
0.1481	0.1515	0.1228	0.0622	0.1545	889	556	717	563	296
0.1481	0.1515	0.1228	0.0622	0.1586	947	574	740	573	316
0.1031	0.1575	0.1364	0.0662	0.1621	887	564	759	597	332
0.1031	0.1575	0.1364	0.0662	0.1645	887	561	756	591	349
0.1031	0.1575	0.1364	0.0662	0.1628	963	575	775	578	359
0.1031	0.1575	0.1364	0.0662	0.1672	1088	591	796	598	365
0.0795	0.1729	0.1502	0.0693	0.1657	1050	625	791	636	361
0.0795	0.1729	0.1502	0.0693	0.1674	1084	647	820	695	382
0.0795	0.1729	0.1502	0.0693	0.1662	1101	655	830	694	390
0.0795	0.1729	0.1502	0.0693	0.1701	1194	678	860	706	400
0.1052	0.1793	0.1539	0.0735	0.1798	1127	692	858	748	392
0.1052	0.1793	0.1539	0.0735	0.1816	1203	705	873	731	418
0.1052	0.1793	0.1539	0.0735	0.1798	1255	719	891	726	442
0.1052	0.1793	0.1539	0.0735	0.1838	1468	744	922	760	442

SERIES 131 - 140

617	4253	60.1	311.1	75.6	107.4	84.3	0.933	0.923	0.979
627	4531	61.8	345.9	72.6	111.9	90.4	0.933	0.923	0.979
627	4492	57.9	292.2	70.2	102.0	87.3	0.940	0.930	0.992
653	4719	64.7	321.0	76.8	120.3	94.8	0.947	0.930	1.035
638	4543	67.6	325.5	78.9	125.7	92.5	0.982	0.947	1.154
647	4759	68.6	349.8	75.6	124.2	89.4	0.982	0.947	1.154
662	4774	64.9	327.0	73.8	109.2	87.6	0.997	0.966	1.150
699	5002	66.5	343.5	72.0	116.4	88.7	0.989	0.966	1.106
691	4818	66.4	352.8	74.1	108.6	88.2	0.985	0.969	1.062
704	4935	62.3	344.7	60.9	104.7	82.1	0.978	0.963	1.049
706	5006	66.8	343.2	59.7	101.4	87.3	0.978	0.969	1.019
730	5243	67.6	346.5	59.7	117.3	89.9	0.970	0.963	1.005
720	4793	68.5	350.7	60.0	106.2	88.2	0.966	0.965	0.972
736	5191	73.5	393.9	61.8	108.6	92.2	0.966	0.965	0.972
746	5159	71.5	339.3	60.0	112.8	90.8	0.959	0.959	0.959
773	5489	79.4	383.4	66.6	127.2	102.8	0.981	0.977	0.998
759	5329	78.2	417.9	69.0	123.9	99.0	0.988	0.985	1.002
775	5563	81.8	417.3	63.6	120.3	100.7	0.988	0.985	1.002
787	5749	75.9	335.1	63.0	106.8	99.0	0.988	0.985	1.002
826	5814	80.7	384.0	65.4	120.6	106.2	0.988	0.991	0.971
806	5672	86.8	414.4	67.7	119.9	104.9	0.988	0.996	0.945
814	5878	81.8	426.7	60.3	121.1	103.9	0.998	1.006	0.955
825	5918	77.8	393.8	57.9	106.0	102.8	0.998	1.006	0.955
851	6056	83.4	397.9	60.9	119.9	110.2	0.998	1.006	0.955
854	5921	84.2	414.5	62.2	117.6	100.7	0.998	1.016	0.906
870	6274	87.2	447.7	63.4	123.5	112.3	0.998	1.006	0.955
878	6251	82.5	389.3	57.8	114.0	108.1	0.998	1.016	0.906
905	6437	87.3	418.4	62.8	121.1	114.5	1.007	1.016	0.964
881	6072	87.4	469.1	62.8	116.4	109.1	1.017	1.026	0.974
895	6612	94.0	473.7	64.0	119.9	118.9	1.017	1.026	0.974
903	6523	90.8	422.2	62.8	111.6	121.1	1.027	1.036	0.983
935	7031	97.6	469.6	66.6	125.9	130.0	1.037	1.046	0.993
963	6730	103.3	495.4	68.5	123.5	126.6	1.037	1.046	0.993
988	7174	105.0	491.4	71.8	133.3	136.4	1.047	1.046	1.051
1007	7138	101.6	413.1	65.2	119.6	132.4	1.057	1.066	1.012
1033	7506	108.5	469.6	72.5	137.0	144.3	1.057	1.066	1.012
1034	7218	108.7	487.3	69.1	130.5	136.8	1.067	1.076	1.021
1052	7609	114.0	522.7	71.1	150.0	148.5	1.067	1.086	0.972
1066	7707	109.1	482.8	69.1	140.7	139.5	1.077	1.096	0.982
1097	8012	113.4	541.3	69.8	150.0	159.3	1.077	1.106	0.933
1078	7681	115.1	569.0	66.4	140.1	161.9	1.096	1.116	1.000
1095	7975	115.8	546.7	62.2	140.9	145.8	1.116	1.136	1.019
1100	8024	118.4	532.8	64.2	136.4	160.1	1.116	1.146	0.970
1115	8253	134.4	589.3	71.7	163.5	172.6	1.116	1.146	0.970

SERIES 141 - 150

1.074	0.952	0.867	0.951	0.934	1.051	1.011	1.069	1.006	0.939
1.074	0.961	0.873	0.947	0.934	1.058	1.003	1.082	1.006	0.949
1.076	0.969	0.893	0.947	0.926	1.038	0.992	1.058	1.006	0.949
1.078	0.969	0.899	0.943	0.918	1.064	0.987	1.099	1.014	0.959
1.089	0.981	0.910	0.962	0.937	1.087	0.990	1.130	1.029	0.978
1.084	0.990	0.910	0.966	0.953	1.080	0.990	1.121	1.021	0.988
1.064	0.990	0.936	0.970	0.953	1.048	0.985	1.076	1.029	0.988
1.084	1.007	0.936	0.970	0.953	1.035	0.977	1.061	1.014	0.988
1.037	1.012	0.952	0.977	0.953	0.987	0.972	0.994	1.006	0.998
1.039	1.004	0.945	0.973	0.945	0.981	0.969	0.986	0.998	0.988
1.037	1.004	0.956	0.965	0.953	0.987	0.977	0.992	0.991	0.978
1.039	0.987	0.958	0.953	0.945	0.987	0.982	0.990	0.991	0.978
1.028	0.981	0.962	0.934	0.961	0.973	0.977	0.971	0.983	0.978
1.030	0.973	0.968	0.927	0.953	0.967	0.982	0.960	0.983	0.978
1.028	0.948	0.968	0.919	0.961	0.973	0.982	0.969	0.975	0.978
1.028	0.948	0.995	0.927	0.961	0.992	0.990	0.993	0.991	0.998
0.987	0.956	1.002	0.952	0.991	0.982	0.992	0.978	0.991	1.008
0.993	0.988	0.995	0.968	0.966	0.976	0.992	0.969	0.991	1.008
0.993	0.997	0.989	0.976	0.966	0.970	0.992	0.960	0.983	1.008
0.987	0.988	0.989	0.972	0.991	0.976	0.990	0.970	0.983	1.008
0.971	0.992	0.993	1.007	0.999	0.957	0.982	0.946	0.983	1.018
0.962	0.973	1.004	1.017	1.019	0.976	0.992	0.969	0.993	1.008
0.943	0.973	1.024	1.007	1.019	0.957	0.992	0.941	0.983	1.008
0.943	0.973	1.024	0.997	1.029	0.957	0.992	0.941	0.993	1.008
0.933	0.992	1.024	1.007	1.019	0.957	0.972	0.950	0.993	1.008
0.914	1.002	1.024	1.017	1.009	0.957	0.972	0.950	0.983	1.008
0.914	0.992	1.034	1.017	1.019	0.957	0.992	0.941	0.983	1.008
0.924	0.992	1.044	1.007	1.029	0.957	0.992	0.941	0.983	1.008
0.933	0.992	1.044	1.027	1.049	0.976	0.992	0.969	0.983	1.008
0.943	0.992	1.054	1.027	1.049	0.986	0.992	0.983	0.983	0.998
0.952	0.992	1.064	1.038	1.049	0.986	0.992	0.983	0.993	0.998
0.971	1.002	1.064	1.038	1.060	1.005	1.002	1.006	1.003	1.008
0.962	1.011	1.064	1.058	1.049	1.024	1.011	1.030	1.013	1.018
0.962	1.021	1.074	1.078	1.049	1.014	1.021	1.011	1.013	1.018
0.962	1.041	1.074	1.078	1.070	1.014	1.041	1.002	1.013	1.027
0.971	1.070	1.084	1.088	1.060	1.024	1.061	1.007	1.023	1.037
0.981	1.080	1.094	1.098	1.070	1.024	1.061	1.007	1.023	1.037
0.990	1.089	1.094	1.108	1.090	1.024	1.061	1.007	1.023	1.037
1.000	1.109	1.104	1.088	1.100	1.014	1.070	0.989	1.023	1.047
1.000	1.109	1.114	1.098	1.110	1.024	1.080	0.999	1.023	1.047
1.000	1.148	1.124	1.108	1.120	1.033	1.100	1.004	1.033	1.057
1.028	1.177	1.144	1.118	1.140	1.053	1.120	1.023	1.033	1.057
1.028	1.186	1.154	1.118	1.140	1.043	1.139	1.000	1.043	1.067
1.009	1.157	1.164	1.118	1.160	1.043	1.139	1.000	1.043	1.067

SERIES 151 - 160

1.076	1.0253	1.0192	0.9981	0.9414	1.0368	1.0216	0.0412	0.9542	1.2168
1.065	1.0253	1.0192	0.9981	0.9414	1.0368	1.0216	0.0420	0.9542	1.2168
1.065	1.0253	1.0192	0.9981	0.9414	1.0368	1.0216	0.0442	0.9542	1.2168
1.071	1.0253	1.0192	0.9981	0.9414	1.0368	1.0216	0.0464	0.9542	1.2168
1.082	1.0821	1.0507	1.0755	0.9527	1.1141	1.0704	0.0407	0.9462	1.1841
1.056	1.0581	1.0141	1.0515	0.9454	1.1395	1.0465	0.0447	0.9462	1.1841
1.072	1.0224	0.9662	1.0161	0.9713	1.1035	1.0113	0.0489	0.9462	1.1841
1.040	0.9800	0.9133	0.9740	0.9696	1.0274	0.9693	0.0553	0.9462	1.1841
1.014	0.9598	0.9265	0.9713	0.9607	0.9924	0.9790	0.0504	0.9491	0.9902
1.009	0.9723	0.9866	0.9840	0.9559	0.9480	0.9917	0.0480	0.9491	0.9902
1.003	0.9733	0.9896	0.9850	0.9624	0.9342	0.9927	0.0457	0.9491	0.9902
1.003	0.9781	1.0161	0.9899	0.9745	0.8940	0.9976	0.0457	0.9491	0.9902
0.987	0.9810	1.0334	0.9832	0.9713	0.6835	0.9587	0.0430	0.9353	0.9171
0.987	0.9771	0.9896	0.9793	0.9721	0.9395	0.9549	0.0454	0.9353	0.9171
0.971	0.9810	0.9896	0.9632	0.9680	0.9639	0.9587	0.0436	0.9353	0.9171
0.983	0.9896	1.0090	0.9919	0.9777	0.9533	0.9671	0.0447	0.9353	0.9171
0.972	0.9916	0.9998	0.9981	0.9818	0.9618	0.9608	0.0501	0.9402	0.8428
0.972	0.9877	0.9805	0.9906	0.9658	0.9829	0.9626	0.0528	0.9402	0.8428
0.957	0.9713	0.9733	0.9789	0.9650	0.9491	0.9728	0.0572	0.9402	0.8428
0.957	0.9733	0.9662	0.9761	0.9801	0.9819	0.9829	0.0546	0.9402	0.8428
0.946	0.9704	0.9560	0.9917	0.9866	0.9946	0.9986	0.0611	0.9647	0.9706
0.978	0.9781	0.9540	0.9963	0.9987	1.0083	1.0059	0.0634	0.9647	0.9706
0.958	0.9704	0.9408	0.9853	1.0012	0.9808	1.0023	0.0733	0.9647	0.9706
0.978	0.9646	0.9459	0.9688	0.9996	0.9639	0.9967	0.0707	0.9647	0.9706
0.978	0.9800	0.9805	0.9716	1.0060	0.9713	0.9903	0.0681	1.0045	0.9353
0.958	0.9723	0.9672	0.9615	1.0141	0.9681	0.9857	0.0661	1.0045	0.9353
0.958	0.9540	0.9438	0.9560	1.0125	0.9533	0.9654	0.0614	1.0045	0.9353
0.958	0.9675	0.9744	0.9551	1.0149	0.9670	0.9645	0.0593	1.0045	0.9353
0.958	0.9829	1.0141	0.9633	1.0157	0.9967	0.9700	0.0613	1.0093	0.9367
0.968	1.0079	1.0660	0.9780	1.0173	1.0094	0.9737	0.0558	1.0093	0.9367
0.988	0.9829	0.9408	0.9880	1.0173	1.0147	0.9857	0.0536	1.0093	0.9367
0.998	1.0234	1.0660	0.9981	1.0214	1.0665	1.0023	0.0579	1.0093	0.9367
1.008	1.0311	1.0558	1.0054	1.0359	1.0877	1.0124	0.0631	1.1028	0.9758
1.008	1.0176	1.0344	1.0119	1.0481	1.0411	1.0133	0.0647	1.1028	0.9758
0.998	1.0320	1.0589	1.0228	1.0545	1.0400	1.0161	0.0643	1.1028	0.9758
1.008	1.0455	1.0731	1.0375	1.0772	1.0147	1.0318	0.0648	1.1028	0.9758
1.008	1.0369	1.0650	1.0641	1.0820	0.9946	1.0603	0.0684	1.1938	1.0305
1.008	1.0369	1.0344	1.0815	1.0990	0.9946	1.0594	0.0702	1.1938	1.0305
0.998	1.0272	1.0131	1.0714	1.0990	0.9956	1.0613	0.0741	1.1938	1.0305
0.998	1.0388	1.0344	1.0705	1.1192	1.0210	1.0410	0.0692	1.1938	1.0305
1.008	1.0638	1.0538	1.0750	1.1597	1.0189	1.0438	0.0730	1.2131	1.0729
1.008	1.0850	1.0568	1.0750	1.2082	1.0453	1.0585	0.0754	1.2131	1.0729
1.018	1.0687	1.0650	1.0796	1.1799	1.0348	1.0603	0.0833	1.2131	1.0729
1.018	1.0609	1.0548	1.0668	1.1815	1.0051	1.0631	0.0760	1.2131	1.0729

APPENDIX C COMPUTATION

This appendix presents copies of the specifications of the two general computer programs that were specially written for the study. The first of these, A TSLS, estimates the parameters of a set of simultaneous equations by the method of two stage least squares, as described in chapter 4. The second, A NRSS, obtains the solution of a set of nonlinear simultaneous equations by the Newton Raphson iterative method, as described in chapter 5. The specifications and programs are published by the Edinburgh Regional Computing Centre.

A TSLS

ct Number 224

fication 00.018.210

ge Atlas Autocode

e Edinburgh KDF 9, compiler AA (or ABC)

029 - ISO cards

Michael Allingham, Department of Economics, University of Edinburgh.

1 October 1968

is This program estimates the parameters of a set of simultaneous equations by the method of two stage least squares (TSLS).

ption The method of TSLS estimates parameters without simultaneous equation bias by replacing all explanatory endogenous variables in the structural equations by their estimated values in terms of all the predetermined variables, from the least squares estimates of the reduced forms, and applying least squares to the modified structural equations.

This program follows a condensed method of the above, fully treated in, inter alia, Johnston (pages 258 -260). The matrix $(\underline{X}' \underline{X})^{-1}$, where $\underline{X}$ is the matrix of the predetermined variables, is formed for the model, then $\underline{X}(\underline{X}' \underline{X})^{-1} \underline{X}'$ is postmultiplied in each equation by the matrix of the explanatory endogenous variables to give the modified explanatory endogenous variables, which are combined in a partitioned matrix $\underline{A}$ with the matrix of the predetermined variables appearing in the equation, giving the explanatory side of the modified structural equation; $(\underline{A}' \underline{A})^{-1} \underline{A}'$ is then postmultiplied by the vector of the dependent variable to give the vector of the TSLS estimates of the parameters. The vector of residuals is then formed in the usual way, and used to calculate a statistic corresponding to the multiple correlation coefficient, von Neumann ratio, and residual variance. Finally, $(\underline{A}'\ \underline{A})^{-1}$ is multiplied by the latter to give the asymptotic variance-covariance matrix for the estimators, and thus their (asymptotic) standard errors. Before commencing, all variables are transformed into the deviations about their means to reduce rounding errors; thus there is no explicit constant term.

The main criteria considered in writing the program were efficiency in the utilisation of space (more than time) and compactness; accordingly, the program is slightly inefficient in its use of time, and to counteract this, only produces the more important information about the estimates.

y facilities

The routines print, unit k, invert k, matrix div k, matrix copy k, matrix mult k, matrix mult k′, matrix trans k and the real function sqrt are called from the permanent material.

ctions

The program occupies 3299 (3017) words, and the number of locations declared is:

$$n^2 + h^2 + f^2 + ng + 2nh + nf + g + h + 2f + 9,$$

where: n = number of observations,

g = number of endogenous variables in the model,

h = number of predetermined variables in the model,

f = maximum number of explanatory variables in an equation.

Thus the above expression should not exceed (approximately) 10,000 for any model. In many cases this (non-stringent) condition may be avoided by estimating more than one model.

presentation For each model:

number of endogenous variables,

number of predetermined variables,

number of observations,

number of equations,

the data, variable by variable, with all the predetermined variables first (the variables then assume the identifying numbers given by the order in which they were read, from 1 to $(g + h)$);

for each equation:

number of explanatory endogenous variables,

number of predetermined variables,

the identifying numbers of the explanatory endogenous variables,

the identifying numbers of the predetermined variables,

the identifying number of the dependent variable;

terminator:

the last model is followed by '-1'.

The input is in free format; it is desirable that all variables be scaled so that they lie within the interval $(10^{-3}, 10^{3})$ in modulus.

The compiling time is 39 sec/35 sec; the running time for the example below was 2 sec/1 sec.

: First line:

title, number of endogenous variables, predetermined variables, observations, and equations;

<u>for each equation</u>:

the equation number, identifying number of the dependent variable, standard error of estimate, von Neumann ratio, and statistic corresponding to the multiple correlation coefficient - this being (var(y) - var(u) - cov(y*, u)) / var(y), or equivalently (var(y*) + cov(y*u)) / var(y), where y is the dependent variable, y* its estimated value, and u the estimated residual.

<u>for each explanatory variable</u>:

its identifying number, coefficient, standard error, mean elasticity, and modulus of the t-ratio

<u>Note</u>

Only the coefficient of the constant term is given.
If the mean of the dependent variable is less than 10^{-6} in modulus, a caption '(MNZ)' is printed instead of the mean elasticities. All integers are in integer form, real numbers depending on the units used are in floating point form to 4 significant figures, and those not dependent on units are in fixed point form to 3 decimal places.

le

To estimate the parameters of the model:

$$y_{1,t} = a_{1,1} + a_{1,2}\, y_{2t} + u_{1,t}$$

$$y_{3,t} = a_{2,1} + a_{22}\, y_{2t} + a_{23}\, x_{1,t} + u_{2,t}$$

$$\boxed{y_{2t} = y_{1,t} + y_{3,t} + x_{2,t}}$$

from the (suitably scaled) data:

x_1	x_2	y_1	y_2	y_3
1.02	2.36	1.09	1.44	1.08
1.08	2.47	1.12	1.48	1.12
1.12	3.17	1.11	1.53	1.05
1.05	3.36	1.10	1.54	0.98
0.98	3.40	1.15	1.59	1.07
1.07	3.38	1.20	1.67	1.28
1.28	3.32	1.24	1.72	1.47
1.47	3.41	1.25	1.75	1.59
1.59	3.32	1.28	1.78	1.67
1.67	2.89	1.31	1.77	1.71

This example is taken from Johnston (pages 268-272), with slight changes in the notation; for example $x_{1,t} \equiv y_{3,t-1}$

The input starts with the model parameters:

3 2 10 2

followed by the data punched (column by column) in the order above, followed by the parameters for the first equation:

1 0 4 3

then the second:

1 1 4 1 5

and finally the terminator,

-1

The output, which is self explanatory, from this input is given below:

ALLINGHAM TSLS: 3 EVS 2 PVS 10 OBS 2 EQS

EQN 1 DEP 3 SRE 2.260$_{10}$ -2 VRR 0.955 RSQ 0.906

VAR	CFT	STR	ETY	MDT
4	6.022$_{10}$ -1	6.254$_{10}$ -2	0.827	9.629
CON	2.059$_{10}$ -1			

EQN 2 DEP 5 SRE 8.833$_{10}$ -2 VRR 0.945 RSQ 0.912

VAR	CFT	STR	ETY	MDT
4	3.374$_{10}$ -1	5.110$_{10}$ -1	0.422	0.660
1	9.082$_{10}$ -1	2.420$_{10}$ -1	0.860	3.753
CON	-3.668$_{10}$ -1			

e indications If the number of predetermined variables exceeds the number of observations, in any model, the program will stop (at line 2). If any equation to be estimated is not identified, that is if less predetermined variables are excluded from the relationship than explaratory endogenous variables are included, or is not determined, that is if there are more explanatory variables than observations, the program will again stop (at line 14). When functioning correctly the program stops at line 2 (all line numbers as per listing below, starting from line 1).

onal comments Models non-linear in variables but linear in (unknown) parameters may be estimated by this program be re-defining the combined variables; the program is not suitable for estimating models non-linear in parameters.

Large models with more predetermined variables than observations may be estimated by division into blocks, deciding which predetermined variables are to be taken account of in each block, and treating each block as a separate model.

This program may also be used to estimate the parameters of a model by the method of ordinary least squares (OLS), by specifying that all (even the 'dependent') variables are predetermined. Thus the example above may be estimated by OLS by specifying the model parameters

0 5 10 2

and the equation parameters

0 1 4 3

0 2 4 1 5

The program is not however designed to be efficient for large scale OLS estimation.

A listing of the program is given (page 8) to facilitate modification to suit individual needs. For example the main data may be read from magnetic tape (tape 1, section j) by deleting

'read (x(t, i));'

from line 8, and inserting

'claim tape (1); read (j); read from file

(1, j, x(1, 1), x(n, h + g)); release tape (1)'

after line 6.

nce Johnston, J. Econometric Methods McGraw Hill, New York, 1963.

```
1'; %INTEGER G,N,I,J,K,L,M,T; %REAL R

EAD (G); %STOP %IF G=-1; READ (H,N,L); %STOP %IF H%G N %AND G%G C
NPAGE; %CAPTION ALLINGHAM%U TSLS;; PRINT (G,6,0); %CAPTION %U EVS
INT (H,6,0); %CAPTION %U PVS; PRINT (N,6,0); %CAPTION %U OBS
INT (L,6,0); %CAPTION %U EQS; NEWLINES (10)
EGIN; %ARRAY X(0:N,1:H+G),A(1:H,1:H)

  %CYCLE I=1,1,H+G; X(0,I)=0
  %CYCLE T=1,1,N; READ (X(T,I)); X(0,I)=X(0,I)+X(T,I); %REPEAT
  X(0,I)=X(0,I)/N; %CYCLE T=1,1,N; X(T,I)=X(T,I)-X(0,I); %REPEAT
  %REPEAT; ->%G 2 %IF G=0
  %BEGIN; %ARRAY B(1:H,1:N),C(1:H,1:H)

     MATRIX TRANS K (B,X,H,N); MATRIX MULT K (C,B,X,H,N,H)
     INVERT K (A,C,H,R); %END

; %CYCLE J=1,1,L; READ (G,K); %STOP %IF G+K%G N-3 %OR H-K%L G
  %BEGIN; %INTEGERARRAY E(1:G+K)

     %ARRAY B(1:G+K,1:G+K),C(1:G+K,1:1),D(1:N,1:G+K)
     %BEGIN; %ARRAY U(1:N,1:H),V(1:N,1:N); ->%G 1 %IF G=0

        %CYCLE I=1,1,G; READ (L); E(I)=L
        %CYCLE T=1,1,N; D(T,I)=X(T,L); %REPEAT
        %REPEAT
        MATRIX MULT K (U,X,A,N,H,H); MATRIX MULT K' (V,U,X,N,H,N)
        MATRIX MULT K (U,V,D,N,N,G); MATRIX TRANS K (V,U,G,N)
        ->%G 2 %IF K=0
     1: %CYCLE I=1,1,K; READ (L); E(G+I)=L
        %CYCLE T=1,1,N; V(G+I,T)=X(T,L); D(T,G+I)=X(T,L); %REPEAT
        %REPEAT
     2: READ (L); %CYCLE T=1,1,N; U(T,1)=X(T,L); %REPEAT
        MATRIX MULT K (C,V,U,G+K,N,1); MATRIX TRANS K (U,V,N,G+K)
        MATRIX MULT K (B,V,U,G+K,N,G+K); %END

     %BEGIN; %ARRAY U(1:G+K,1:G+K),V(1:G+K,1:1)

        INVERT K (U,B,G+K,R); MATRIX COPY K (B,U,G+K,G+K)
        MATRIX MULT K (V,U,C,G+K,G+K,1); MATRIX COPY K (C,V,G+K,1); %END

     %BEGIN; %REAL P,Q,S; %ARRAY U(1:N,1:1)

        MATRIX MULT K (U,D,C,N,G+K,1); P=0; Q=0; R=0; S=0
        %CYCLE T=1,1,N; P=P+(X(T,L)-U(T,1)-X(T-1,L)+U(T-1,1))**2 %IF T%G 1
        Q=Q+X(T,L)**2; R=R+(X(T,L)-U(T,1))**2; S=S+U(T,1)*(X(T,L)-U(T,1))
        %REPEAT
        %CYCLE I=1,1,G+K; U(I,1)=SQRT(R*B(I,I)/(N-G-K-1)); %REPEAT
        %CAPTION EQN; PRINT (J,2,0); SPACES (7); %CAPTION DEP
        PRINT (L,2,0); %CAPTION %U %U %U SRE; PRINT FL (SQRT(R/(N-G-K-1)),3)
        %CAPTION %U %U %U VNR; PRINT (P/R,1,3); %CAPTION %U %U %U RSQ
        PRINT ((Q-R-S)/Q,1,3); NEWLINES (2); %CAPTION VAR; SPACES (13)
        %CAPTION CFT; SPACES (13); %CAPTION STR; SPACES (9); %CAPTION ETY
        SPACES (9); %CAPTION NDT; NEWLINE; P=X(0,L)
        %CYCLE I=1,1,G+K; P=P-C(I,1)*X(0,E(I))
        PRINT (E(I),2,0); SPACES (6); PRINT FL (C(I,1),3); SPACES (6)
        PRINT FL (U(I,1),3); ->%G 1 %IF MOD(X(0,L))%L .000001
        PRINT (C(I,1)*X(0,E(I))/X(0,L),7,3); ->%G 2
     1: SPACES (7); %CAPTION (MNZ)
     2: PRINT (MOD(C(I,1)/U(I,1)),7,3); NEWLINE
        %REPEAT
        %CAPTION CON; SPACES (6); PRINT FL (P,3); NEWLINES (5); %END; %END

  %REPEAT; NEWPAGE; %END

G 1: %ENDOFPROGRAM
```

MA

A NRSS

act number 298

ification 00.018.445

age Atlas Autocode

ne Edinburgh KDF 9, compiler AA (or ABC)

029 - ISO cards

r Michael Allingham, Department of Economics, University of Edinburgh

1 January 1969

sis This program obtains the solutions of a set of non-linear simultaneous equations by the Newton-Raphson iterative method (NR).

iption The NR method obtains a set of solutions by expanding each equation in a Taylor's series about some initial approximation, and obtaining the next approximation as the value of this series truncated after the first derivative term. This may be interpreted in multi-dimensional space as the approximation of the hypersurface representing each non-linear equation by its hyperplane that is tangential at the initial approximate solution point, the next approximation being given by the intersection of these hyperplanes.

Expressing the n equations in matrix form by

$$f_i(\underline{x}) = 0 \qquad i = 1, 2, \ldots n,$$

where $f_i(\underline{x})$ is shorthand notation for $f_i(x_1, \ldots x_n)$, the new trial solution $\underline{x}^{(k+1)}$ is then given in terms of the old $\underline{x}^{(k)}$ by

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} - \left[\underline{J}^{(k)}\right]^{-1} \underline{u}^{(k)}$$

where $J^{(k)}$ is the Jacobian $\left[\frac{\partial f_i}{\partial x_j}\right]$ for $\underline{x} = \underline{x}^{(k)}$, and $u_i^{(k)} = f_i\left[\underline{x}^{(k)}\right]$

(i, j = 1, 2, ...n); the new solution vector is thus obtained by subtracting the error given by the old multiplied by the inverse

Jacobian at that stage, from the old.

This program follows a similar pattern, but introduces (where necessary due to non-convergence) a damping factor $g(g \geqslant 1)$ to decrease the adjustment made at each stage, so

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} - \frac{1}{g}\left[\underline{J}^{(k)}\right]^{-1} u^{(k)}$$

At first g is set to 1; $\underline{x}^{(1)}$ is then set to $\underline{x}^{(0)}$, the given initial approximate solution, and $\underline{u}^{(1)}$ (or $f_i(\underline{x}^{(1)})$) is formed. Next the quantities $f_i(x_1^{(1)}, \ldots (1 + e)x^{(1)}, \ldots x_n^{(1)})$, where e is a small positive number, are calculated, and $\underline{J}^{(1)}$ is formed using the approximation

$$\frac{\partial f_i}{\partial x_j} = \frac{1}{ex_j^{(1)}} \left[f_i(x_1^{(1)}, \ldots (1 + e)x_j^{(1)}, \ldots x_n^{(1)}) - f_i(\underline{x}^{(1)})\right]$$

at stage 1. $\underline{J}^{(1)}$ is then inverted and post-multiplied by $\underline{u}^{(1)}$, and the resulting vector is subtracted from $\underline{x}^{(0)}$ to give $\underline{x}^{(2)}$. Now $\underline{u}^{(2)}$ is formed, and this trial solution is tested for convergence, this being defined as occuring where every element in $\underline{u}$ is less in absolute value than some small positive number h. If convergence is achieved the results are printed; if not the process is repeated. At the third and later stages a test is made for divergence, which is defined as occuring where the sum of squared errors increases with successive iterations, that is

$$\underline{u}^{(k+1)'} \underline{u}^{(k+1)} > \underline{u}^{(k)'} \underline{u}^{(k)}$$

If this happens the factor g is doubled, $\underline{x}^{(1)}$ is again set to $\underline{x}^{(0)}$, and the process is repeated. If divergence still occurs when $g = 64$ (that is 2^6) the method fails and the program stops. The quantity e is arbitrarily set at 10^{-6}; h is determined by the input of an integer m, h being 10^{-m}.

y facilities The routines print, unit k, invert k, matrix div k, matrix mult k, and matrix trans k are called from the permanent material.

ictions

The program (without equations) occupies 2736 (2743) words, and $2(n + 1)^2 + 9$ locations are declared; thus size is not an immediate constraint, as approximately 75 equations may be solved, and the rounding errors involved in the inversion of $\underline{J}$ are likely to become unacceptable at around this number of equations - 50 is a safer maximum.

Any type of non-linear function acceptable to the compiler may be used as long as there is no chance of its argument falling outside an acceptable range (such as using log x where $x < 0$).

The program is not suitable for systems with any root very near zero, though such systems may readily be solved indirectly by replacing the original variable, x_i ($x_i \simeq 0$), by a new variable, $x_i + p$ (p being an arbitrary non-zero number), and adding p to the resultant value of x.

The NR method is very powerful when the trial solution is near the true solution, but if not it is unlikely to achieve convergence at all. This is partially overcome by the damping factor, but none the less care should be exercised in selecting initial values for highly non-linear systems.

ntation

Data:

n, m, $x_1^{(0)}, x_2^{(0)} \dots x_n^{(0)}$ in free format;

Equations:

The equations to be solved must be inserted between line 24 ('comment begin equations') and line 28 ('comment end equations'). These must be in the form

$$\underline{u(i)} = f_i(\underline{x}) \qquad i = 1, 2, \dots n$$

where the $f_i(\underline{x})$ are written in a form acceptable to the compiler, containing only constants and the variables x_j, which must be written as x(j, 1). (Double underlining indicates actual input; this section may be made clearer by the example below).

The compiling time (for the program without equations inserted) is 29 sec/26 sec; the running time for the example below was 1 sec/ 0 sec.

t

First line:

Title, number of equations, n, degree of accuracy, m, damping factor required , g, and number of iterations required (at this g);

for each variable:

the identifying number, solved value, initial value, and percentage difference between these, that is 100 ('solved' - 'initial')/'initial'; this last figure would be of use if the equations constituted a model for forecasting x and the initial values where the 'true' values of x.

Note

All integers are in integer form, real numbers depending on the units used are in floating point form to 4 significant figures, and those not dependent on units are in fixed point form to 3 decimal places.

le

To solve the 3 equation system:

$$x_1 + x_2^2 + x_3^3 = 32$$

$$x_1/x_2 = (x_2 + x_3 - 1)^{-\frac{1}{2}}$$

$$\log x_1 + \log x_2 + \log x_3 = 1.79176$$

to an accuracy of 10^{-3} using the initial values 0.9, 2.2, 2.7. (The solution is 1, 2, 3)

The data input is

3 3 0.9 2.2 2.7

and the equations to be inserted are written as:

```
u(1) = x(1, 1) + x(2, 1)**2 + x(3, 1)**3 - 32
u(2) = x(1, 1)/x (2, 1) - 1/(sqrt(x(2, 1) + x(3, 1) - 1))
u(3) = log (x(1, 1)) + log (x(2, 1)) + log (x(3, 1)) -
                                                   1.79176
```

The output for this example is given below:

ALLINGHAM NRSS: EQS 3 ACC 3 FAC 1 ITN 4

VAR	SOL	INL	PDF
1	1.000⏨ 0	9.000⏨ -1	11.111
2	2.000⏨ 0	2.200⏨ 0	-9.091
3	3.000⏨ 0	2.700⏨ 0	11.111

re
tions

If the method fails to converge with a damping factor of up to 2^6 the program will stop at line 6; if any element in $\underline{x}^{(k)}$ becomes near zero (less than 10^{-24} in modulus) the program will stop at line 9. When functioning correctly the program stops at line 26. (all line numbers as per listing below, starting from line 1, and not counting the inserted equations).

onal
ts

This program may of course be used (inefficiently) for solving linear systems, when the solution will be obtained in one iteration - since the hyperplanes are then 'exact approximations' of the hypersurfaces; similarly very rapid convergence is achieved if the functions are approximately linear in the region containing the solution and the initial trial values. If the functions are approximately quadratic in this region convergence is also rapid:

$$u_i^{(k+1)} = u_i^{(k)} \times 10^{-k}.$$

In general acceptable convergence will usually be reached after approximately 5 iterations (this is with no damping, that is $g = 1$), if it is reached at all.

A listing of the program is given below to facilitate modification to suit individual needs, such as changing the value of the derivative increment, e, or the maximum value of the convergence factor, g. This listing contains the equations used in the example above.

```
N: %INTEGER I,J,K,L,M,N: %REAL A,B,C,R,S

AD (N,M): B=10**36: C=2: S=10**(-M)
EGIN: %ARRAY U(1:N),W(1:N,1:1),X(1:N,0:1),V,Y(1:N,1:N)

 %CYCLE I=1,1,N: READ (X(I,0)): %REPEAT
 %CYCLE I=1,1,N: X(I,1)=X(I,0): %REPEAT
 L=0: C=C/2: %STOP %IF C%L 1/100
 K=0: A=0: L=L+1: -%G 1
 %CYCLE I=1,1,N: W(I,1)=U(I): K=1 %IF MOD(U(I))%G S
 A=A+U(I)**2: %STOP %IF MOD(X(I,1))%L 10**(-24): %REPEAT
 -%G 5 %IF K=0: -%G 7 %IF A%G B: B=A
 %CYCLE I=1,1,N: X(I,1)=(1+S)*X(I,1): -%G 1
 %CYCLE J=1,1,N: V(J,I)=(U(J)-W(J,1))/(S*X(I,1)): %REPEAT
 X(I,1)=X(I,1)/(1+S): %REPEAT
 INVERT K(Y,V,N,R): MATRIX MULT K(V,Y,W,N,N,1)
 %CYCLE I=1,1,N: %CYCLE J=1,1,N: V(I,J)=C*V(I,J): %REPEAT: %REPEAT
 MATRIX SUB K(X,X,V,N,1): -%G 4
 NEWPAGE: %CAPTION ALLINGHAM%U NRSS:%U %U %U EQS: PRINT (N,1,0)
 %CAPTION %U %U %U ACC: PRINT (M,1,0): %CAPTION %U %U %U FAC
 PRINT (1/C,1,0): %CAPTION %U %U %U ITN: PRINT (L,1,0): NEWLINES (10)
 %CAPTION VAR: SPACES (13): %CAPTION SOL: SPACES (13): %CAPTION INL
 SPACES (9): %CAPTION PDF: %CYCLE I=1,1,N: NEWLINE: PRINT (I,2,0)
 SPACES (6): PRINT FL (X(I,1),3): SPACES (6): PRINT FL (X(I,0),3)
 PRINT (100X(I,1)/X(I,0)-100,7,3): %REPEAT: NEWPAGE: -%G 6
    %V  BEGIN EQUATIONS
        U(1)=X(1,1)+X(2,1)**2+X(3,1)**3-32
        U(2)=X(1,1)/X(2,1)-1/(SQRT(X(2,1)+X(3,1)-1))
        U(3)=LOG(X(1,1))+LOG(X(2,1))+LOG(X(3,1))-1.79176
    %V  END EQUATIONS: -%G 3 %IF K=1: -%G 2
 %END: %ENDOFPROGRAM
```

MA

APPENDIX D ESTIMATES

This appendix presents the numerical estimates of the model in tabular form, giving coefficients and (below) t ratios for major explanatory variables; coefficients alone are given for the seasonal terms (QU, QD, QT) and the constant term (CN). Also reported are the von Neumann ratio (VN), goodness of fit (GF), standard error of estimate (SE), and (in the industry models) any outlying (at the one percent level) observations measured from 1956 Q1 (OL). Coefficients and standard errors are given to three significant figures (subject to a maximum of six decimal places), the t and von Neumann ratios to one decimal place, and the goodness of fit to two decimal places. These concepts are discussed in chapter 4.

The members of the following sets of variables are distinguished for convenience, but are only meaningful in aggregate: $[s_{\underline{e}}, s_{\underline{v}}, s_{\underline{a}}]$, $[s_{\underline{p}}, s_{\underline{o}}]$, $[p_{\underline{e}}, p_{\underline{v}}, p_{\underline{a}}, p_{\underline{o}}]$. $p_{\underline{n}}$ is defined by $p_{\underline{n}} = P$ $(= p_{\underline{m}})$. The following variables are exogenous: TY, TZ, $tc_{\underline{f}}$, $tc_{\underline{v}}$, $tc_{\underline{t}}$, $tc_{\underline{o}}$, GG, GL, GT, GK, RB, HP, $WT_{\underline{n}}$, $WT_{\underline{m}}$, QU, QD, QT.

ESTIMATES OF THE INDUSTRY MODELS

Equation. 1:. demand.　　　Dependant variable. X.p

OL	Ind	Explanatory variables				QU	QT	VN	
		g.p	$s - s_{-1}$	fdc 1+	fdc 2+	QD	CN	GF	SE
	n	6.81	0.000017	0.00117	-0.000206	-0.0188	0.0548	1.3	0.0128
		23.2	0.4	3.3	2.9	0.0157	-0.0540	.99	
	f	39.5	0.000156	0.000055		0.00240	-0.0351	1.5	0.0157
		6.0	1.3	0.5		0.0197	0.266	.99	
	c	28.1	0.000877	0.000203		0.0996	0.00110	0.8	0.0255
		7.9	1.3	0.9		0.0315	-0.251	.98	
	i	22.2	0.00619	-0.000732	0.000408	0.0366	-0.0531	2.1	0.0467
		2.2	4.4	1.4	0.8	0.0228	0.0245	.95	
	e	26.3		0.000327	-0.000287	0.0325	-0.0455	2.0	0.0148
		11.6		3.3	2.4	-0.00326	-0.0455	.99	
	v	22.5	0.000399	0.000833	0.000558	0.0129	-0.0853	2.0	0.0328
		2.0	0.6	3.4	1.4	-0.0223	0.0706	.98	
	a	17.1	0.00113	0.000253		0.0316	-0.0536	1.0	0.0257
		3.9	1.9	0.8		0.0110	0.468	.96	
	t	41.1	0.000711	-0.000591		0.0278	0.0163	1.0	0.0150
		16.0	3.1	6.8		-0.00184	-0.0709	.99	
29	p	11.3	0.00293	0.000114	0.000387	0.0316	-0.0837	2.0	0.0270
		1.3	3.0	1.3	1.5	-0.00549	0.120	.98	
29	o	21.5	0.00246	0.000339		-0.000980	-0.0132	1.2	0.0311
		5.9	2.3	1.2		-0.00138	-0.0293	.98	

+ these are:

ind	fdc 1	fdc 2	
n	N	M	(N,M apply to industry n)
f	c		(N,M apply to industry m)
c	GG		"
i	M	GK	"
e	I	M	"
v	c	N	"
a	GK		"
t	M		"
p	c	N	"
o	N		"

ESTIMATES OF THE INDUSTRY MODELS

Equation. 2;. supply. Dependant variable. X.

OL Ind	Explanatory variables				QU	QT	VN	SE
	ł	k_{-1}ł/(ł + u)			QD	CN	GF	
n	0.000084	0.000051			-0.0941	-0.00998	2.0	0.0214
	1.4	9.2			-0.0211	-1.69	.98	
f	0.000475	0.000386			-0.0490	-0.0556	1.5	0.0135
	2.4	29.2			0.0149	-0.0759	.98	
c+								
i	0.00298	0.000116			0.0155	-0.0811	0.5	0.0565
	5.4	4.3			0.0184	-1.04	.79	
e	0.000139	0.000432			-0.0230	-0.114	1.3	0.0235
	1.2	8.0			-0.0400	-0.323	.97	
v	0.00112	0.000541			0.0521	-0.106	0.9	0.0496
	2.5	9.6			0.0575	-0.775	.85	
a	0.000374	0.000191			0.0125	-0.0815	0.8	0.0373
	1.0	2.8			0.00585	0.550	.63	
t	0.000320	0.00128			0.000699	-0.0849	0.8	0.0287
	2.7	8.4			-0.0361	-2.70	.84	
p	0.000529	0.00101			0.00503	-0.119	0.9	0.0334
	0.8	6.9			-0.00489	-0.467	.94	
o	0.00165	0.000353			-0.0341	-0.0401	0.6	0.0352
	2.8	4.2			0.0150	-0.953	.93	

+ not applicable: ł = $ł_{-1}$

ESTIMATES OF THE INDUSTRY MODELS

Equation. 3: stocks. Dependant variable. $s - s_{-1}$

OL	Ind	Explanatory variables				QU	QT	VN	SE
		$x.p - x_{-1}p_{-1}$	p/p_{-1}	RB	$s_{-1} - s_{-2}$	QD	CN	GF	
	n	1050			0.124	138	-13.8	2.1	64.6
		2.7			0.7	-56.8	-13.1	.25	
	f	257	1060	-234		49.1	36.4	2.7	21.9
		0.9	3.1	0.6		-2.65	-1060	.57	
38	c		293		0.0782	-14.8	-1.18	2.1	9.15
			1.4		0.4	1.19	-285	.32	
19	i	63.8			0.458	8.80	21.1	2.1	8.14
		1.7			2.8	10.9	-7.74	.30	
	e	198	106		0.395	22.5	36.0	2.4	9.72
		2.7	0.5		2.3	27.4	-125	.46	
	v	12.4	168		0.424	-4.83	1.06	2.4	10.6
		0.4	0.7		2.2	2.41	-162	.28	
	a	51.9	305		0.355	-1.93	6.99	2.4	10.4
		0.8	1.3		1.9	5.07	-302	.32	
37	t		809	-377	0.0866	1.12	-28.4	2.2	13.4
			2.2	1.6	0.5	3.37	-786	.62	
	p		564	-68.8	0.327	0.448	-0.607	1.8	4.90
			3.1	0.7	1.9	0.592	-560	.45	
	o		163		0.321	0.490	-1.70	2.1	5.64
			1.2		1.7	-1.41	-161	.24	

ESTIMATES OF THE INDUSTRY MODELS

Equation 4^A (labour)^A Dependant variable $(ł + u)^A$

OL	Ind	Explanatory variables				QU	QT	VN	SE
		e/E	L	U	+	QD	CN	GF	
	n	2400	0.498			-89.3	-9.08	0.4	108
		1.1	7.6			-17.9	688	.84	
	f		0.0176	0.101		-26.1	12.0	0.4	11.6
			3.2	3.3		-15.5	400	.69	
	c+	48.4	0.0204	0.174	0.854	3.58	5.24	0.8	8.94
		1.0	2.7	2.5	3.2	-0.0755	-137	.44	
	i	183	0.0362			16.6	-11.8	0.7	12.0
		2.0	4.8			5.70	-437	.53	
29	e	200	0.331	0.314		64.5	11.5	1.8	27.0
		0.8	15.2	3.9		32.5	-5740	.98	
	v	86.4	0.0282			15.1	-6.10	0.4	20.4
		0.9	2.8			0.765	130	.24	
	a		0.0326			4.93	-10.1	0.3	17.6
			4.1			-4.83	57.2	.39	
	t	2330				58.9	-39.4	1.1	49.3
		4.8				9.61	-502	.48	
29	p		0.0675	0.0895		9.00	4.40	1.5	6.52
			21.8	5.1		5.17	-954	.96	
	o	193	0.0643	0.0307		13.8	-7.79	1.0	9.08
		1.1	9.6	1.3		-1.51	-722	.92	

\+ extra variable included for industry <u>c</u> is: ł/u

Equation 4^B: (labour)^B: $(ł + u)^B_i = 1.01\ ł_i$

Equation 4: labour: $(ł + u)_i = \max \left[(ł + u)^A_i,\ (ł + u)^B_i \right]$

for i = all industries

Equation. 5:. capital   Dependant variable. i.

OL	Ind	Explanatory variables: $x.p - x_{-4}p_{-4}$	lyp^{+}	k_{-1}	i_{-1}	QU / QD	QT / CN	VN / GF	SE
29	n		0.0800	0.0118	0.499	-28.1	-29.4	1.8	25.0
			1.5	1.0	3.2	-42.9	-157	.97	
	f		0.0797	-0.00405	0.318	-1.26	-0.105	1.7	1.63
			2.7	0.4	1.9	0.0962	5.27	.96	
	c		0.0947	-0.0113	0.742	-10.3	-8.37	2.1	3.42
			3.4	2.5	7.2	-5.71	20.1	.86	
	i	6.12	0.0825	-0.00807	0.834	8.59	16.0	2.9	5.78
		0.5	2.9	2.7	9.1	12.1	-4.45	.86	
	e	10.0	0.0927	-0.00244	0.142	-10.8	-8.31	2.1	2.41
		0.7	3.5	0.5	0.8	-9.44	14.7	.96	
	v		0.0140	0.00901	0.560	-5.33	-1.74	2.4	3.26
			0.5	1.2	4.0	-0.375	-4.12	.69	
	a		0.0249	0.00169	0.597	-4.94	-2.49	2.1	0.986
			1.9	0.7	4.3	-3.00	3.79	.85	
	t	12.2	0.0449	0.0116	0.676	-4.03	-2.53	1.9	2.47
		1.4	1.6	0.4	4.6	-0.489	-29.6	.93	
	p		0.0359		0.644	-4.18	-4.15	2.2	1.77
			2.7		5.0	-3.05	5.10	.79	
	o	4.78	0.105	0.0153		-4.09	-2.58	2.1	1.90
		0.7	5.6	3.3		-2.97	-8.68	.96	

$^{+}$ $lyp = S_i(z_{-i}(1 - TZ_{-i}))$, $i = 1, 2, 3, 4$

ESTIMATES OF THE INDUSTRY MODELS

Equation.6:.price..   Dependant variable.p.

OL	Ind	Explanatory variables				QU	QT	VN	SE
		h	1.e/x	s_{-1}/x_{-1}		QD	CN	GF	
	n+								
	f	0.966				0.00676	-0.00272	0.6	0.0157
		18.1				0.00336	0.0293	.91	
	c	0.239	0.000338			0.00449	-0.00155	0.2	0.0126
		3.7	0.5			0.00330	0.730	.34	
	i	0.585	0.000422			-0.00303	-0.00344	0.8	0.0108
		5.4	1.1			-0.00241	0.373	.87	
5	e	1.44	0.000187	-0.000103		-0.0133	-0.0104	1.1	0.00634
		32.6	1.7	2.3		-0.00944	-0.406	.99	
	v	1.30				0.000824	-0.000008	0.9	0.00393
		125.6				0.00112	-0.299	.99	
	a	1.35	0.00151	-0.000054		-0.00173	-0.0183	1.1	0.00857
		11.7	3.5	1.2		0.000903	-0.502	.99	
	t	0.887		-0.000032		-0.00167	-0.000160	0.3	0.00526
		42.5		1.1		-0.00117	0.136	.98	
	p	0.731				0.00159	-0.000509	0.2	0.00864
		20.0				-0.000335	0.267	.93	
	o	1.23	0.00152	-0.000111		-0.0172	-0.00576	1.7	0.00552
		17.7	4.6	1.4		0.00148	-0.415	.99	

+ not applicable: p = P

ESTIMATES OF THE INDUSTRY MODELS

Equation. 7: earnings Dependant variable. e/e_{-1}

OL	Ind	Explanatory variables				QU	QT	VN	SE
		$1/u$ +	x/x_{-1}	z/z_{-4}	B/B_{-4}	QD	CN	GF	
	n			0.158	0.496	0.0439	-0.0409	2.5	0.0440
				1.6	1.0	-0.0648	0.350	.54	
	f	0.241		0.0588		0.0429	-0.0376	2.5	0.0330
		0.6		0.5		-0.0455	0.949	.57	
	c +	5.96				-0.0249	-0.0545	3.0	0.0401
		0.5				-0.00711	1.02	.23	
	i	0.0344	0.182		0.246	0.0295	0.0214	2.8	0.0179
		0.5	1.7		1.1	-0.00237	0.563	.47	
	e			0.160	0.279	-0.00454	-0.115	2.7	0.0529
				1.1	0.5	-0.0932	0.615	.53	
	v					-0.0406	0.0170	2.8	0.0676
						0.0838	1.00	.33	
	a	0.0975		0.0410	0.188	0.00771	-0.0117	2.7	0.0195
		0.4		2.1	0.7	-0.0178	0.775	.42	
	t	0.222	0.102			0.0244	-0.00619	2.9	0.0145
		1.5	1.1			-0.00721	0.898	.57	
16	p		1.06			0.0584	0.0333	2.8	0.0714
			2.0			-0.00517	-0.0779	.63	
	o		0.455		0.226	0.0524	0.00296	2.7	0.0300
			1.8		0.6	-0.0360	0.319	.44	

+ for industry c this variable is: $1/U$

ESTIMATES OF THE INDUSTRY MODELS

Equation. 8b: profits (b)   Dependant variable. z.

OL Ind	Explanatory variables				QU	QT	VN	SE
	x.p	±.e/x	$x_{-4}p_{-4}$		QD	CN	GF	
n	702	0.0728	-144		-2.56	3.88	1.0	33.5
	3.7	0.8	0.6		-19.5	-242	.95	
f	82.4	0.539			-12.0	-5.80	1.7	5.74
	3.1	1.8			-7.90	-38.1	.92	
c	129	0.884	-54.5		-17.4	-2.14	1.7	5.64
	4.4	2.1	1.9		-11.6	-74.3	.90	
i	70.3	-0.844	47.8		-14.9	18.4	1.5	8.95
	4.3	2.8	1.5		-8.49	54.8	.61	
e	144	0.138	-86.3		-18.1	2.40	1.2	7.87
	4.0	1.0	1.9		-7.90	19.7	.90	
v	64.0		-29.3		-9.56	2.96	2.8	11.5
	2.0		0.8		-8.95	15.4	.39	
a	66.3	-0.974	82.3		12.2	20.2	2.7	11.7
	1.4	1.7	0.8		-1.36	9.00	.38	
t	111	0.293	-80.8		-11.3	-4.78	2.4	7.14
	5.7	2.4	3.2		-5.23	-19.5	.70	
p	56.3	-0.470	12.2		-3.10	6.26	1.9	4.27
	3.1	2.4	0.5		-0.569	24.6	.83	
o	91.2	-0.227	-26.0		0.694	3.13	1.9	3.47
	7.5	1.3	1.2		-3.76	14.6	.96	

ESTIMATES OF THE ECONOMY MODEL

(i)

Ind	Explanatory variables				QU	QT	VN	SE
					QD	CN	GF	
	Consumption:	dependent variable c						
	DLI +	b/B	c_{-1}	HP				
f	0.146		0.451		-242	-46.8	1.8	19.1
	5.1		3.9		-21.6	474	.99	
v		-227	0.452	-81.3	72.2	16.0	1.7	24.6
		2.4	2.8	1.8	77.1	274	.84	
t	0.0715	-241			-177	-115	2.3	22.5
	3.5	0.6			-107	455	.95	
o	0.313	-875	0.552		-129	50.8	2.3	33.8
	4.7	1.9	5.3		-5.12	526	.99	
	Consumer prices:	dependent variable b						
	p	tc						
f	1.00	2.00			-0.0147	0.00496	0.9	0.0213
	8.9	2.7			0.00535	-0.305	.92	
v	0.107	0.444			0.00339	0.00605	1.1	0.0145
	2.0	3.9			0.0134	0.807	.45	
t	0.348	2.07			-0.00791	-0.00370	1.1	0.00650
	5.7	11.9			-0.00499	0.558	.98	
o	1.58				-0.00307	-0.00929	0.3	0.0217
	25.1				-0.0131	-0.574	.96	

+ DLI = (L.E + GL + GT)(1 - TY)

ESTIMATES OF THE ECONOMY MODEL

(ii)

Dep var (ind)	Explanatory variables (ind)				QU QD	QT CN	VN GF	SE
	Exports							
	WT (n)	N_{-1} (n)	WT (m)					
N (n)	80.8	0.523			-26.0	-16.9	2.7	7.20
	4.2	4.4			-22.2	8.04	.94	
N (m)			479		36.3	-11.3	1.6	17.4
			34.0		29.3	286	.98	
	Imports							
	TFI +	M_{-1} (n)	M_{-1} (m)					
M (n)	0.0167	0.736			-16.9	-38.5	1.2	28.7
	2.1	6.1			-26.0	129	.86	
M (m)	0.0437		0.560		-0.772	-10.5	1.6	19.8
	3.3		4.0		9.78	-103	.97	
	Other income							
	L.E	Z						
V	0.181	0.133			-22.2	-4.89	1.3	17.3
	9.9	2.8			-10.3	86.8	.99	
	Indirect taxation							
	TCT ++							
A	1.15				25.2	-11.0	2.2	30.9
	28.2				-21.4	51.2	.96	

+ TFI = L.E + Z + V + GL

++ TCT = $\Sigma_i(c_i tc_i)$, for i = industries f, v, t, o

DETERMINISTIC RELATIONS: INDUSTRY MODELS

8a. Profits (a) $\quad z_i = q_i x_i p_i - l_i e_i;$

9. Intermediate demand $\quad g_i = S_j(x_j w_j a_{ij});$

10. Materials price $\quad h_i = S_j(p_j a_{ji})/S_j(a_{ji});$

where

$$\left[a_{ij}\cdot 10^4\right] = \begin{bmatrix} 1432 & 2807 & 2455 & 1795 & 1079 & 975 & 1263 & 1203 & 1842 & 1914 \\ 173 & 1132 & 130 & 0 & 0 & 0 & 0 & 6 & 14 & 0 \\ 308 & 621 & 2513 & 429 & 231 & 193 & 140 & 198 & 402 & 629 \\ 173 & 40 & 148 & 3289 & 1448 & 1247 & 2278 & 6 & 69 & 75 \\ 203 & 171 & 195 & 204 & 1495 & 533 & 322 & 153 & 201 & 215 \\ 168 & 3 & 0 & 4 & 16 & 1863 & 11 & 3 & 0 & 22 \\ 144 & 232 & 231 & 157 & 567 & 904 & 1553 & 99 & 35 & 199 \\ 88 & 81 & 69 & 14 & 99 & 110 & 38 & 4231 & 118 & 672 \\ 333 & 343 & 170 & 11 & 96 & 28 & 48 & 54 & 2438 & 210 \\ 376 & 198 & 163 & 89 & 307 & 544 & 285 & 131 & 48 & 1129 \end{bmatrix},$$

$$\left[q_i\cdot 10^4\right] = \begin{bmatrix} 2407 & 209 & 177 & 183 & 487 & 224 & 167 & 234 & 146 & 192 \end{bmatrix},$$

$$\left[w_i\cdot 10^4\right] = \begin{bmatrix} 6065 & 452 & 357 & 357 & 878 & 415 & 336 & 483 & 289 & 368 \end{bmatrix}.$$

DETERMINISTIC RELATIONS: ECONOMY MODEL

Aggregate relations

$X = S_i(w_i x_i),$ $\quad P = S_i(w_i x_i p_i)/X,$

$L = S_i(l_i),$ $\quad E = S_i(l_i e_i)/L,$

$C = S_i(c_i),$ $\quad B = S_i(c_i b_i)/C,$

$I = S_i(i_i),$ $\quad S = S_i(s_i),$

$U = S_i(u_i),$ $\quad Z = S_i(z_i),$

$N = N_{\underline{n}} + N_{\underline{m}},$ $\quad M = M_{\underline{n}} + M_{\underline{m}}.$

Accounting relations

$Y^e = C + I + GG + GL + GK + N - M + S - S_{-1} - A,$

$Y^i = L.E + GL + Z + V.$

(All subscripts i,j refer to all relevant industries or categories.)

APPENDIX E SOLUTIONS

This appendix presents the numerical solutions of the model.

The first part concerns the solutions of the individual industry models (using true values of the link variables) for each quarter of 1966. For each industry the percentage errors (that is 100('predicted' - 'true')/'true' values) of the predictions of output (X) and profits (Z) are given for each basic model (MA and MB) and for the three naive models (NS, NL, NG). The industry results are summarised in terms of the aggregate (AGG) and root-mean-square (RMS) figures.

The second part concerns the iterative solution of the model (that is MA) as a whole, giving information on the solutions (using naive lagged values of the link variables as initial values) after each of fifty iterations for each quarter of 1966. The degree of convergence (CVE), consistency (CSY), and accuracy is given for each iteration (ITN); accuracy is indicated by the percentage error of the prediction of mean gross domestic product (Y GDP), and for the output (X) and profits (Z) of each industry summarised in aggregate (AGG) and root-mean-square (RMS) terms.

The concepts mentioned above are discussed in chapter 5; all figures are given to one place of decimals.

SOLUTIONS OF THE INDUSTRY MODELS: QUARTER ONE

(USING TRUE VALUES OF LINK VARIABLES)

IND	VAR	PERCENTAGE ERROR FOR MODEL				
		NS	NL	NG	MA	MB
N	X	7.6	7.3	7.5	1.3	1.3
	Z	13.2	13.4	13.4	17.6	-10.6
F	X	4.2	10.0	10.4	-1.0	-1.0
	Z	1.5	3.1	3.1	1.7	-25.2
C	X	-5.4	-2.4	-2.3	0.8	0.8
	Z	13.1	20.2	20.7	-6.0	32.[illegible]
I	X	-0.6	8.1	9.0	-0.1	-0.1
	Z	-14.0	0.0	2.7	-27.4	-36.6
E	X	-1.3	11.5	13.5	-3.8	-3.8
	Z	3.8	1.9	1.9	-1.8	16.7
V	X	-5.6	9.9	12.9	-1.9	-1.9
	Z	29.2	-16.7	-4.7	29.3	155.8
A	X	-1.8	8.2	9.4	-2.6	-2.6
	Z	-31.7	-3.2	17.4	-9.6	-27.4
T	X	-0.8	7.8	8.6	-1.8	-1.8
	Z	22.5	54.9	66.6	8.3	2.8
P	X	-3.2	8.5	10.2	-2.7	-2.7
	Z	3.3	26.2	32.8	-13.6	-46.6
O	X	1.4	7.6	8.0	-1.2	-1.2
	Z	7.1	14.3	14.8	2.0	-16.3
AGG	X	4.0	7.8	8.3	0.1	0.1
	Z	7.6	11.7	14.1	6.4	-3.9
RMS	X	3.9	8.5	9.6	2.0	2.0
	Z	17.3	21.9	25.8	15.2	55.5

SOLUTIONS OF THE INDUSTRY MODELS: QUARTER TWO

(USING TRUE VALUES OF LINK VARIABLES)

IND	VAR	PERCENTAGE ERROR FOR MODEL				
		NS	NL	NG	MA	MB
N	X	-4.8	-12.0	-11.5	0.9	0.9
	Z	-1.3	-14.3	-12.8	13.7	44.2
F	X	-5.5	-9.5	-9.3	-1.8	-1.8
	Z	-2.3	-3.8	-3.7	-0.2	-8.6
C	X	1.9	7.4	7.7	0.6	0.6
	Z	7.6	-6.5	-4.9	20.5	16.8
I	X	3.1	3.8	3.8	-3.0	-3.0
	Z	19.2	35.9	38.6	-7.6	-20.4
E	X	4.7	6.0	6.1	-3.7	-3.7
	Z	-2.5	-6.2	-6.1	-2.0	41.1
V	X	2.2	7.9	8.3	-2.0	-2.0
	Z	0.0	-29.2	-22.6	22.0	90.3
A	X	5.8	7.7	7.8	1.5	1.5
	Z	110.0	176.7	207.7	44.9	58.8
T	X	4.1	4.9	4.9	-2.1	-2.1
	Z	0.0	-22.5	-18.4	7.2	-37.4
P	X	1.3	4.6	4.7	-2.6	-2.6
	Z	27.1	22.9	23.0	24.9	0.1
O	X	0.0	-1.4	-1.4	0.1	0.1
	Z	4.5	-3.0	-2.5	6.1	24.4
AGG	X	-2.1	-6.1	-5.7	-0.1	-0.1
	Z	4.1	-3.8	-1.7	10.4	28.3
RMS	X	3.8	7.1	7.1	2.1	2.1
	Z	36.5	58.9	68.0	19.7	42.5

SOLUTIONS OF THE INDUSTRY MODELS: QUARTER THREE

(USING TRUE VALUES OF LINK VARIABLES)

IND	VAR	PERCENTAGE ERROR FOR MODEL NS	NL	NG	MA	MB
N	X	-3.2	1.4	1.7	-1.6	-1.6
	Z	2.6	3.9	3.9	22.8	41.8
F	X	5.8	11.6	12.0	0.0	0.0
	Z	-5.7	-3.5	-3.5	-1.1	-28.4
C	X	2.4	0.5	0.6	-0.3	-0.3
	Z	-3.2	-10.5	-10.0	16.2	18.3
I	X	14.8	11.3	11.4	-8.9	-8.9
	Z	36.8	10.5	14.8	-0.6	-46.4
E	X	4.9	0.0	0.2	-2.5	-2.5
	Z	1.3	3.8	3.8	7.3	29.6
V	X	16.8	14.3	14.4	-0.8	-0.8
	Z	-17.2	-17.2	-17.2	15.5	1.7
A	X	8.4	2.1	2.4	2.9	2.9
	Z	-23.1	-107.7	-63.4	-26.3	-27.9
T	X	8.0	3.6	3.8	-0.9	-0.9
	Z	16.4	16.4	16.4	1.2	-50.8
P	X	13.6	12.0	12.1	3.3	3.3
	Z	-2.0	-28.6	-22.9	13.1	-33.4
O	X	2.1	2.1	2.1	1.0	1.0
	Z	-1.5	-5.9	-5.7	13.7	1.9
AGG	X	0.7	2.8	3.0	-1.4	-1.4
	Z	1.3	-2.9	-1.2	13.5	14.9
RMS	X	9.5	8.0	8.1	3.3	3.3
	Z	15.9	36.4	23.5	14.6	32.3

SOLUTIONS OF THE INDUSTRY MODELS: QUARTER FOUR

(USING TRUE VALUES OF LINK VARIABLES)

IND	VAR	PERCENTAGE ERROR FOR MODEL				
		NS	NL	NG	MA	MB
N	X	-2.1	1.9	1.1	-0.1	-0.1
	Z	-10.3	-12.6	-12.6	15.2	29.4
F	X	-4.0	-9.6	-9.3	0.9	0.9
	Z	3.7	9.6	9.9	9.4	-18.8
C	X	-4.2	-6.5	-6.5	-0.5	-0.5
	Z	-5.9	-3.0	-2.9	16.0	16.9
I	X	-5.8	-19.8	-17.9	27.4	27.4
	Z	-17.4	-47.8	-39.6	37.6	5.4
E	X	-12.3	-16.7	-16.5	-4.4	-4.4
	Z	-3.0	-4.3	-4.3	6.3	41.3
V	X	-2.5	-18.9	-16.5	4.2	4.2
	Z	34.9	58.1	63.0	47.1	129.2
A	X	-1.0	-9.3	-8.7	7.7	7.7
	Z	-30.4	-14.3	-9.5	-23.4	-16.5
T	X	-4.2	-11.8	-11.3	2.0	2.0
	Z	-23.7	-36.2	-34.5	-0.2	29.2
P	X	-5.0	-17.9	-16.4	9.9	9.9
	Z	-24.6	-23.1	-23.0	-15.8	-65.2
O	X	-1.4	-3.4	-3.4	5.4	5.4
	Z	-10.5	-9.2	-9.2	5.4	3.4
AGG	X	-3.5	-4.1	-3.8	1.2	1.2
	Z	-9.1	-10.2	-9.3	11.5	20.0
RMS	X	5.3	13.1	12.2	9.9	9.9
	Z	19.7	28.4	27.8	22.5	50.3

SOLUTIONS AFTER SUCCESSIVE ITERATIONS: QUARTER ONE

ITN	CVE	CSY	ACCURACY				
			Y GDP	X AGG	X RMS	Z AGG	Z RMS
1	16.3	3.6	5.5	3.8	4.5	11.3	13.3
2	49.9	3.4	4.9	1.5	12.0	7.5	19.1
3	12.3	3.1	3.6	-0.7	13.6	4.8	22.3
4	6.3	2.8	2.2	-2.6	14.5	2.5	23.9
5	3.5	2.6	0.8	-4.7	15.2	0.2	25.0
6	1.9	2.3	-0.6	-6.5	16.0	-1.9	25.9
7	1.1	2.0	-2.0	-8.1	16.8	-3.9	26.7
8	1.1	1.7	-3.4	-9.6	17.6	-5.8	27.6
9	1.5	1.4	-4.8	-10.9	18.3	-7.6	28.4
10	2.0	1.1	-6.1	-12.1	19.0	-9.3	29.3
11	2.1	0.8	-7.5	-13.2	19.6	-11.0	30.2
12	2.3	0.5	-8.7	-14.2	20.2	-12.6	31.1
13	2.4	0.3	-10.0	-15.1	20.8	-14.1	32.2
14	2.4	0.1	-11.1	-15.9	21.2	-15.6	33.3
15	2.1	-0.2	-12.2	-16.5	21.7	-17.0	34.3
16	1.6	-0.3	-13.2	-17.2	22.1	-18.2	35.2
17	0.8	-0.4	-13.9	-17.5	22.5	-19.3	36.0
18	0.4	-0.5	-14.5	-18.4	22.8	-20.4	36.7
19	0.5	-0.6	-15.1	-18.9	23.2	-21.4	37.4
20	0.6	-0.6	-15.7	-19.3	23.6	-22.3	38.1
21	0.6	-0.7	-16.2	-19.8	23.9	-23.3	38.8
22	0.6	-0.7	-16.7	-20.2	24.3	-24.2	39.4
23	0.6	-0.8	-17.2	-20.7	24.6	-25.0	40.1
24	0.6	-0.8	-17.7	-21.1	25.0	-25.8	40.7
25	0.6	-0.9	-18.1	-21.5	25.4	-26.6	41.3
26	0.6	-0.9	-18.6	-21.9	25.7	-27.4	41.9
27	0.6	-0.9	-19.0	-22.3	26.1	-28.2	42.4
28	0.6	-1.0	-19.4	-22.6	26.4	-28.9	43.0
29	0.6	-1.0	-19.8	-23.0	26.8	-29.6	43.5
30	0.6	-1.0	-20.2	-23.4	27.2	-30.3	44.0
31	0.6	-1.1	-20.6	-23.7	27.5	-31.0	44.5
32	0.6	-1.1	-21.0	-24.1	27.9	-31.7	45.0
33	0.5	-1.1	-21.4	-24.4	28.3	-32.3	45.5
34	0.5	-1.2	-21.8	-24.8	28.6	-33.0	45.9
35	0.5	-1.2	-22.1	-25.1	29.0	-33.6	46.3
36	0.5	-1.2	-22.5	-25.4	29.3	-34.2	46.7
37	0.5	-1.2	-22.8	-25.7	29.7	-34.7	47.1
38	0.5	-1.3	-23.1	-26.1	30.1	-35.3	47.5
39	0.5	-1.3	-23.5	-26.4	30.4	-35.8	47.9
40	0.5	-1.3	-23.8	-26.7	30.8	-36.4	48.2
41	0.5	-1.3	-24.1	-27.0	31.1	-36.9	48.5
42	0.5	-1.4	-24.4	-27.3	31.5	-37.4	48.8
43	0.5	-1.4	-24.7	-27.6	31.9	-37.8	49.1
44	0.5	-1.4	-25.0	-27.9	32.2	-38.3	49.4
45	0.5	-1.4	-25.3	-28.2	32.6	-38.7	49.7
46	0.5	-1.4	-25.6	-28.5	32.9	-39.1	49.9
47	0.5	-1.4	-25.9	-28.8	33.3	-39.5	50.2
48	0.4	-1.5	-26.2	-29.1	33.7	-39.9	50.4
49	0.4	-1.5	-26.5	-29.4	34.0	-40.2	50.7
50	0.4	-1.5	-26.7	-29.7	34.4	-40.5	50.9

SOLUTIONS AFTER SUCCESSIVE ITERATIONS: QUARTER TWO

ITN	CVE	CSY	ACCURACY					
			Y GDP	X AGG	X RMS	Z AGG	Z RMS
1	80.6	4.2	-4.8	-1.4	2.5.	7.3	17.6
2	3.9	1.6	-5.2	-2.4	10.8	5.8	22.7
3	3.1	-1.2	-6.2	-3.5	13.0	4.0	27.0
4	1.2	-3.7	-7.2	-4.6	14.1	2.2	29.2
5	0.7	-6.2	-8.2	-5.7	14.9	0.5	30.8
6	0.8	-8.5	-9.1	-6.7	15.5	-1.1	32.3
7	1.0	-10.8	-10.1	-7.6	16.0	-2.5	33.6
8	1.2	-12.9	-11.0	-8.5	16.6	-3.9	34.9
9	1.4	-15.0	-11.9	-9.3	17.1	-5.3	36.2
10	1.5	-17.0	-12.8	-10.0	17.5	-6.6	37.5
11	1.6	-18.9	-13.6	-10.7	17.9	-7.8	38.7
12	1.7	-20.6	-14.4	-11.3	18.4	-9.0	39.9
13	1.5	-22.1	-15.1	-11.8	18.7	-10.1	41.0
14	0.9	-23.1	-15.7	-12.3	19.1	-11.1	41.9
15	0.4	-23.9	-16.2	-12.8	19.5	-12.0	42.7
16	0.4	-24.6	-16.7	-13.2	19.8	-12.9	43.3
17	0.5	-25.2	-17.2	-13.7	20.2	-13.8	43.9
18	0.5	-25.8	-17.6	-14.1	20.5	-14.6	44.4
19	0.5	-26.3	-18.1	-14.4	20.9	-15.5	44.9
20	0.5	-26.8	-18.5	-14.8	21.3	-16.2	45.4
21	0.5	-27.2	-18.8	-15.2	21.6	-17.0	45.8
22	0.5	-27.6	-19.2	-15.5	22.0	-17.7	46.2
23	0.5	-28.0	-19.6	-15.8	22.3	-18.5	46.6
24	0.5	-28.4	-20.0	-16.2	22.7	-19.2	47.0
25	0.5	-28.8	-20.3	-16.5	23.0	-19.8	47.4
26	0.5	-29.1	-20.6	-16.8	23.4	-20.5	47.8
27	0.5	-29.4	-21.0	-17.1	23.7	-21.1	48.1
28	0.5	-29.7	-21.3	-17.4	24.1	-21.7	48.5
29	0.5	-30.0	-21.6	-17.7	24.4	-22.3	48.8
30	0.5	-30.3	-21.9	-18.0	24.8	-22.9	49.1
31	0.5	-30.6	-22.2	-18.2	25.1	-23.5	49.4
32	0.4	-30.8	-22.5	-18.5	25.4	-24.0	49.7
33	0.4	-31.1	-22.8	-18.8	25.8	-24.5	50.0
34	0.4	-31.3	-23.1	-19.0	26.1	-25.1	50.3
35	0.4	-31.5	-23.3	-19.3	26.5	-25.6	50.6
36	0.4	-31.7	-23.6	-19.6	26.8	-26.0	50.8
37	0.4	-31.9	-23.9	-19.8	27.1	-26.5	51.1
38	0.4	-32.1	-24.1	-20.1	27.5	-26.9	51.4
39	0.4	-32.3	-24.4	-20.3	27.8	-27.4	51.6
40	0.4	-32.4	-24.6	-20.5	28.1	-27.8	51.9
41	0.4	-32.6	-24.9	-20.8	28.5	-28.2	52.1
42	0.4	-32.7	-25.1	-21.0	28.8	-28.5	52.3
43	0.4	-32.9.	-25.4	-21.3	29.1	-28.9	52.6
44	0.4	-33.0	-25.6	-21.5	29.4	-29.2	52.8
45	0.4	-33.1	-25.8	-21.7	29.8	-29.6	53.1
46	0.4	-33.2	-26.0	-21.9	30.1	-29.9	53.4
47	0.4	-33.3	-26.3	-22.2	30.4	-30.1	53.6
48	0.4	-33.4	-26.5	-22.4	30.7	-30.4	53.9
49	0.4	-33.4	-26.7	-22.6	31.0	-30.6	54.3
50	0.4	-33.5	-26.9	-22.8	31.4	-30.8	54.6

SOLUTIONS AFTER SUCCESSIVE ITERATIONS: QUARTER THREE

ITN	CVE	CSY	ACCURACY				
			Y GDP	X AGG	X RMS	Z AGG	Z RMS
1	74.4	1.7	1.5	0.9	4.0	16.6	22.6
2	7.4	1.3	0.9	-0.6	9.8	14.2	19.4
3	5.4	0.9	-0.2	-2.2	12.9	11.9	19.6
4	2.5	0.5	-1.5	-3.7	14.5	9.4	22.0
5	1.2	0.1	-2.7	-5.2	15.7	7.2	24.3
6	0.9	-0.4	-4.0	-6.6	16.6	5.2	26.2
7	1.0	-0.8	-5.2	-7.8	17.4	3.3	27.8
8	1.3	-1.1	-6.3	-9.0	18.1	1.6	29.3
9	1.6	-1.5	-7.4	-10.0	18.8	-0.0	30.8
10	1.8	-1.9	-8.5	-11.0	19.4	-1.6	32.3
11	1.9	-2.2	-9.5	-11.9	20.0	-3.0	33.7
12	2.0	-2.5	-10.5	-12.6	20.5	-4.5	35.0
13	1.9	-2.8	-11.5	-13.3	21.0	-5.9	36.0
14	1.6	-3.0	-12.2	-14.0	21.5	-7.1	37.1
15	0.7	-3.2	-12.9	-14.6	21.9	-8.3	38.1
16	0.4	-3.3	-13.5	-15.1	22.3	-9.3	39.0
17	0.5	-3.4	-14.0	-15.6	22.7	-10.3	39.9
18	0.5	-3.5	-14.5	-16.1	23.1	-11.2	40.7
19	0.5	-3.6	-15.0	-16.5	23.5	-12.1	41.6
20	0.5	-3.7	-15.4	-16.9	23.9	-13.0	42.4
21	0.5	-3.8	-15.8	-17.3	24.3	-13.8	43.1
22	0.5	-3.9	-16.3	-17.7	24.7	-14.5	43.9
23	0.5	-3.9	-16.7	-18.0	25.0	-15.3	44.6
24	0.5	-4.0	-17.0	-18.4	25.4	-16.0	45.3
25	0.5	-4.1	-17.4	-18.7	25.7	-16.7	45.9
26	0.4	-4.1	-17.8	-19.0	26.1	-17.4	46.6
27	0.4	-4.2	-18.1	-19.3	26.4	-18.0	47.2
28	0.4	-4.2	-18.4	-19.6	26.8	-18.6	47.8
29	0.4	-4.3	-18.8	-19.9	27.1	-19.2	48.4
30	0.4	-4.3	-19.1	-20.2	27.4	-19.8	48.9
31	0.4	-4.4	-19.4	-20.5	27.7	-20.3	49.5
32	0.4	-4.4	-19.7	-20.7	28.0	-20.8	50.0
33	0.4	-4.5	-19.9	-21.0	28.4	-21.4	50.5
34	0.4	-4.5	-20.2	-21.3	28.7	-21.9	51.0
35	0.4	-4.5	-20.5	-21.5	29.0	-22.3	51.5
36	0.4	-4.6	-20.8	-21.7	29.3	-22.8	52.0
37	0.3	-4.6	-21.0	-22.0	29.6	-23.2	52.5
38	0.3	-4.6	-21.3	-22.2	29.9	-23.7	53.0
39	0.3	-4.7	-21.5	-22.4	30.2	-24.1	53.4
40	0.3	-4.7	-21.7	-22.6	30.4	-24.5	53.9
41	0.3	-4.7	-22.0	-22.8	30.7	-24.9	54.3
42	0.3	-4.7	-22.2	-23.0	31.0	-25.2	54.8
43	0.3	-4.8	-22.4	-23.3	31.3	-25.6	55.2
44	0.3	-4.8	-22.6	-23.4	31.6	-25.9	55.6
45	0.3	-4.8	-22.9	-23.6	31.8	-26.2	56.1
46	0.3	-4.8	-23.1	-23.8	32.1	-26.6	56.5
47	0.3	-4.9	-23.3	-24.0	32.4	-26.8	56.9
48	0.3	-4.9	-23.5	-24.2	32.6	-27.1	57.4
49	0.3	-4.9	-23.6	-24.4	32.9	-27.4	57.8
50	0.3	-4.9	-23.8	-24.6	33.1	-27.7	58.2

SOLUTIONS AFTER SUCCESSIVE ITERATIONS: QUARTER FOUR

ITN	CVE	CSY	ACCURACY				
			Y GDP	X AGG	X RMS	Z AGG	Z RMS
1	204.5	0.2	-0.9	-5.6	6.4	0.9	23.3
2	5.6	-0.0	-1.2	-6.4	12.9	1.3	14.7
3	1.5	-0.4	-2.5	-7.3	14.3	-0.2	16.1
4	1.8	-0.7	-3.7	-8.2	14.5	-1.8	16.2
5	2.0	-1.0	-4.8	-9.0	14.7	-3.3	16.5
6	2.0	-1.2	-5.9	-9.7	14.9	-4.7	17.0
7	1.9	-1.5	-7.0	-10.4	15.2	-6.1	17.8
8	1.8	-1.7	-8.0	-11.0	15.5	-7.4	18.6
9	1.6	-1.9	-9.0	-11.5	15.8	-8.7	19.6
10	1.5	-2.1	-9.9	-12.0	16.1	-9.9	20.6
11	1.3	-2.3	-10.7	-12.4	16.4	-11.1	21.5
12	0.8	-2.3	-11.3	-12.8	16.7	-12.2	22.2
13	0.4	-2.4	-11.9	-13.2	17.0	-13.1	22.9
14	0.5	-2.4	-12.4	-13.5	17.3	-14.0	23.6
15	0.6	-2.5	-12.9	-13.9	17.6	-14.9	24.3
16	0.6	-2.5	-13.3	-14.2	17.9	-15.8	25.0
17	0.6	-2.6	-13.8	-14.5	18.3	-16.6	25.6
18	0.6	-2.6	-14.3	-14.9	18.7	-17.4	26.2
19	0.6	-2.6	-14.7	-15.2	19.1	-18.2	26.8
20	0.6	-2.7	-15.1	-15.5	19.5	-18.9	27.4
21	0.6	-2.7	-15.5	-15.8	19.9	-19.7	27.9
22	0.6	-2.7	-15.9	-16.1	20.3	-20.4	28.4
23	0.6	-2.7	-16.3	-16.4	20.7	-21.1	28.9
24	0.6	-2.8	-16.7	-16.7	21.1	-21.8	29.4
25	0.6	-2.8	-17.1	-17.0	21.6	-22.5	29.8
26	0.6	-2.8	-17.5	-17.3	22.0	-23.1	30.2
27	0.6	-2.8	-17.8	-17.6	22.5	-23.7	30.6
28	0.6	-2.9	-18.2	-18.0	22.9	-24.3	30.9
29	0.6	-2.9	-18.5	-18.3	23.4	-24.9	31.2
30	0.6	-2.9	-18.9	-18.6	23.8	-25.4	31.5
31	0.5	-2.9	-19.2	-18.9	24.3	-26.0	31.8
32	0.5	-2.9	-19.6	-19.2	24.8	-26.5	32.0
33	0.5	-3.0	-19.9	-19.5	25.2	-26.9	32.2
34	0.5	-3.0	-20.2	-19.7	25.7	-27.4	32.4
35	0.5	-3.0	-20.5	-20.0	26.2	-27.8	32.6
36	0.5	-3.0	-20.8	-20.3	26.6	-28.1	32.8
37	0.5	-3.0	-21.1	-20.6	27.1	-28.5	33.0
38	0.5	-3.0	-21.4	-20.9	27.6	-28.8	33.3
39	0.5	-3.0	-21.7	-21.2	28.1	-29.0	33.6
40	0.5	-3.0	-22.0	-21.5	28.5	-29.2	34.0
41	0.5	-3.0	-22.3	-21.8	29.0	-29.3	34.6
42	0.5	-3.0	-22.6	-22.1	29.5	-29.4	35.4
43	0.5	-3.0	-22.8	-22.4	30.0	-29.4	36.6
44	0.5	-3.0	-23.1	-22.7	30.5	-29.3	38.1
45	0.5	-2.9	-23.3	-23.0	31.0	-29.0	40.3
46	0.5	-2.9	-23.5	-23.3	31.5	-28.7	43.3
47	0.5	-2.9	-23.7	-23.6	32.0	-28.1	47.4
48	0.5	-2.8	-23.9	-23.9	32.5	-27.3	53.1
49	0.5	-2.7	-24.1	-24.2	33.0	-26.1	60.8
50	0.5	-2.6	-24.2	-24.5	33.5	-24.5	71.7